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Optimization of the Fuzzy Investment Portfolio under Conditions of Uncertainty

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***Abstract* – The problem of portfolio optimization under uncertainty is considered. For its solution the application of fuzzy sets theory is suggested. Fuzzy portfolio optimization problem is stated; its model is provided and investigated, as well as the algorithm of its solution is presented in the article. The problem of multicriteria fuzzy portfolio optimization is also considered and investigated. This problem includes two main criteria – portfolio profitability and risk. A mathematical model of this problem is constructed, explored and the sufficient conditions for its convexity are obtained. For better estimation of stock profitability, Fuzzy Group Method of Data Handling (FGMDH) for stock price forecasting is suggested. The experimental investigations of the suggested approach are carried out and their results – optimal portfolios based on the projected stock prices are presented, and its efficiency is evaluated.**

***Keywords* – forecasting, fuzzy portfolio, FGMDH, multicriteria optimization, stock prices**

I. INTRODUCTION

The portfolio analysis exists, perhaps, as long, as people think about acceptance of rational decisions connected with the use of the limited resources. However, the occurrence of portfolio analysis can be dated precisely enough having connected it with the publication of pioneer work of Harry Markowitz (Markowitz H. *Portfolio Selection*) in 1952. The model offered in this work, simple enough in essence, has allowed presenting the basic features of the financial market from the point of view of the investor and has supplied the investor with the tool for development of rational investment decisions.

The central problem in the Markowitz theory is the portfolio choice, that is, a set of shares. Thus, for the estimation of both separate shares and their portfolios, two major factors are considered: profitability and risk of shares and their portfolios. The risk, thus, receives a quantitative estimation. The account of mutual correlation dependences between profitability of shares appears to be the essential moment in the theory. This account allows making effective diversification of portfolio, leading to the essential decrease in a portfolio risk in comparison with the risk of the shares included in it. At last, the quantitative characteristic of the basic investment characteristics allows defining and solving a problem of the choice of an optimum portfolio in the form of a quadratic optimization problem.

However, the worldwide market crises in 1997-1998 and in 2000-2001, which resulted in 10 billion dollar losses only for the American investors, have shown that the existing theories of optimization of share portfolios and forecasting of share indices have exhausted themselves, and essential revision of share management methods is necessary.

Thus, in the light of obvious insufficiency of available scientific methods for management of financial assets, the development of fundamentally new theory of management of the financial systems functioning under conditions of essential uncertainty is necessary. The assistance to this theory was rendered by the theory of the fuzzy sets developed about half a century ago by Lofti Zadeh.

The aims of the present research are to study and qualitatively analyse a new approach to management of the stock portfolio, based on the application of the fuzzy set theory; to develop the algorithms implementing the given approach and compare the results of their application with the results obtained using classical probabilistic methods; and to investigate dual and multicriteria optimization portfolio problems.

II. PROBLEM STATEMENT

The aim of the analysis and optimization of an investment portfolio is to explore the area of portfolio optimization and to perform the comparative analysis of the effective portfolios received by means of Markowitz model and fuzzy set model of share portfolio optimization.

Let us consider a share portfolio from N components and its expected behaviour at time interval $[0,T]$. Each of portfolio components is characterized $i=1,...,N$ by the financial profitability r_i.

The holder of a share portfolio – the private investor or the investment company operates the investments, being guided by certain reasons. On the one hand, the investor tries to maximize the profitability. On the other hand, he fixes a maximum permissible risk of the inefficiency of investments. We assume the capital of the investor to be equal to 1.

The problem of optimization of a share portfolio consists in the finding of a vector of share price distribution of papers in a portfolio $x=\{x_i\}$, $i=\overline{1,N}$ of the investor maximizing the income at the set risk level (obviously, that $\sum_{i-1}^{N} x_i = 1$).

Weaknesses of classical Markowitz model are discussed in [1-4].

In the process of practical application of Markowitz model, its drawbacks have been found out:

1. The hypothesis about normality profitability distributions does not prove to be true in practice.

2. Stationarity of price processes is not always valid.

3. At last, the risk of shares is considered to be dispersion or standard deviation of the share prices from the expected value,

i.e., the decrease in profitability of securities in relation to the expected value and the increase in profitability are estimated absolutely equally. Though for the owner of securities these events are absolutely not the same.

These weaknesses of Markowitz theory define the necessity for the use of essentially new approach to define an optimum investment portfolio.

III. Fuzzy Set Portfolio Model

Main principles of the method are presented in [1, 2, 5, 6].

The risk of a portfolio is not its volatility, but possibility that the expected profitability of a portfolio will appear below a certain pre-established value.

- Correlation of assets in a portfolio is not considered and accounted.
- Profitability of each asset is a fuzzy number. Similarly, restriction on the extremely low level of profitability can be both scalar and fuzzy number of any kind. Therefore, to optimize a portfolio may mean, in such a specific case, the requirement to maximize the expected profitability of a portfolio in the period of time T at the fixed risk level of a portfolio.
- Profitability of assets on termination of ownership term is expected to be equal to r and is in the settlement range. For i-th share:

$\overline{r_i}$ is the expected profitability of i-th share;

r_{1i} is the lower border of profitability of i-th share;

r_{2i} is the upper border of profitability of i-th share;

$r_i = \left(r_{1i}, \overline{r_i}, r_{2i}\right)$ – the profitability of i-th share is a triangular fuzzy number.

Then profitability of a portfolio:

$$r = (r_{\min} = \sum_{i=1}^{N} x_i r_{1i}; \overline{r} = \sum_{i=1}^{N} x_i \overline{r_i}; r_{\max} = \sum_{i=1}^{N} x_i r_{2i}), \quad (1)$$

where x_i – the weight of i-th share in a portfolio, and

$$\sum_{i=1}^{N} x_i = 1, \ 0 \le x_i \le 1 . \quad (2)$$

The critical level of portfolio profitability at the moment of T may be fuzzy triangular type number $r^* = \left(r_1^*; \overline{r}^*; r_2^*\right)$.

IV. Mathematical Model of a Fuzzy Optimization Problem

To define the structure of a portfolio, which will provide the maximum profitability at the set risk level, it is required to solve the following problem [1-6]:

$$\{x_{opt}\} = \{x\} \mid r \to \max, \ \beta = const , \quad (3)$$

where r is the profitability, β is the desired risk, vector components x satisfy (2).

As shown in [1-5], the risk degree of the most expected value of a portfolio is defined:

$$\beta = \begin{cases} 0, \ if \ r^* < r_{\min} \\ R\left(1 + \dfrac{1-\alpha_1}{\alpha_1}\ln(1-a_1)\right), \ if \ r_{\min} \le r^* \le \tilde{r} \\ 1-(1-R)\left(1 + \dfrac{1-\alpha_1}{\alpha_1}\ln(1-a_1)\right), \ if \ \tilde{r} \le r^* < r_{\max} \\ 1, \ if \ r^* \ge r_{\max} \end{cases}, (4)$$

where

$$R = \begin{cases} \dfrac{r^* - r_{\min}}{r_{\max} - r_{\min}}, \ if \ r^* < r_{\max} \\ 1, \ if \ r^* \ge \mathrm{r}_{\max} \end{cases}. \quad (5)$$

$$\alpha_1 = \begin{cases} 0, & if \ r^* < r_{\min} \\ \dfrac{r^* - r_{\min}}{\tilde{r} - r_{\min}}, & if \ r_{\min} \le r^* < \tilde{r} \\ 1, & if \ r^* = \tilde{r} \\ \dfrac{r_{\max} - r^*}{r_{\max} - \tilde{r}}, & if \ \tilde{r} < r^* < r_{\max} \\ 0, & if \ r^* \ge r_{\max} \end{cases}$$

The profitability of a portfolio is as follows:

$$r = (r_{\min} = \sum_{i=1}^{N} x_i r_{1i}; \ \tilde{r} = \sum_{i=1}^{N} x_i \tilde{r}_i; \ r_{\min} = \sum x_i r_{2i}),$$

where $\left(r_{1i}, \overline{r_i}, r_{2i}\right)$ – the profitability of i-th security. Thus, we receive the following problem of optimization (6)-(8):

$$\tilde{r} = \sum_{i=1}^{N} x_i \tilde{r}_i \to \max, \quad (6)$$

$$\beta = const, \quad (7)$$

$$\sum_{i=1}^{N} x_i = 1, \ x_i \ge 0, \ i = \overline{1,N} \quad (8)$$

In case of the risk level variation β 3 cases are possible. We will consider each of them in detail.

1. $\beta = 0$

From (4) it is evident that this case is possible when $r^* < \sum_{i=1}^{N} x_i r_{i1}$.

We receive the following problem of linear programming:

$$\tilde{r} = \sum_{i=1}^{N} x_i \tilde{r}_i \to \max, \tag{9}$$

$$\sum_{i=1}^{N} x_i r_{i1} > r^*, \tag{10}$$

$$\sum_{i=1}^{N} x_i = 1,\ x_i \geq 0,\ i = \overline{1,N}. \tag{11}$$

The obtained result of the problem (9)-(11) solution – vector $x = \{x_i\}$ $i = \overline{1,N}$ is the required structure of an optimum portfolio for the given risk level.

2. $\beta = 1$

From (4) it follows that this case is possible when $r^* \geq \sum_{i=1}^{N} x_i r_{i2}$.

3. $0 < \beta < 1$

From (4) it is evident that this case is possible if $\sum_{i=1}^{N} x_i r_{i1} \leq r^* \leq \sum_{i=1}^{N} x_i \tilde{r}_i$, or if $\sum_{i=1}^{N} x_i \tilde{r}_i \leq r^* \leq \sum_{i=1}^{N} x_i r_{i2}$.

a) Let $\sum_{i=1}^{N} x_i r_{i1} \leq r^* \leq \sum_{i=1}^{N} x_i \tilde{r}_i$

Then using (4) - (5) problem (6) - (8) is reduced to the following problem of nonlinear programming:

$$\tilde{r} = \sum_{i=1}^{N} x_i \tilde{r}_i \to \max, \tag{12}$$

$$\left(\left(r^* - \sum_{i=1}^{N} x_i r_{i1} \right) + \left(\sum_{i=1}^{N} x_i \tilde{r}_i - r^* \right) \cdot \ln \left(\frac{\sum_{i=1}^{N} x_i \tilde{r}_i - r^*}{\sum_{i=1}^{N} x_i \tilde{r}_i - \sum_{i=1}^{N} x_i r_{i1}} \right) \right) \cdot$$

$$\cdot \frac{1}{\sum_{i=1}^{N} x_i r_{i2} - \sum_{i=1}^{N} x_i r_{i1}} = \beta, \tag{13}$$

$$\sum_{i=1}^{N} x_i r_{i1} \leq r^*, \tag{14}$$

$$\sum_{i=1}^{N} x_i \tilde{r}_i > r^*, \tag{15}$$

$$\sum_{i=1}^{N} x_i = 1,\ x_i \geq 0,\ i = \overline{1,N}. \tag{16}$$

б) Let $\sum_{i=1}^{N} x_i \tilde{r}_i \leq r^* \leq \sum_{i=1}^{N} x_i r_{i2}$. Then the problem (6) - (8) is reduced to the following problem of nonlinear programming:

$$\tilde{r} = \sum_{i=1}^{N} x_i \tilde{r}_i \to \max, \tag{17}$$

$$\left(\left(r^* - \sum_{i=1}^{N} x_i r_{i1} \right) - \left(r^* - \sum_{i=1}^{N} x_i \tilde{r}_i \right) \cdot \ln \left(\frac{r^* - \sum_{i=1}^{N} x_i \tilde{r}_i}{\sum_{i=1}^{N} x_i r_{i2} - \sum_{i=1}^{N} x_i r_{i1}} \right) \right) \cdot$$

$$\cdot \frac{1}{\sum_{i=1}^{N} x_i r_{i2} - \sum_{i=1}^{N} x_i r_{i1}} = \beta, \tag{18}$$

$$\sum_{i=1}^{N} x_i r_{i2} > r^*, \tag{19}$$

$$\sum_{i=1}^{N} x_i \tilde{r}_i \leq r^*, \tag{20}$$

$$\sum_{i=1}^{N} x_i = 1,\ x_i \geq 0,\ i = \overline{1,N} \tag{21}$$

The R-algorithm of minimization of non-differentiated functions is applied to find the solution to problems (12) - (16) and (17) - (21) [5, 6]. Both problems: (12) - (16) and (17) - (21) are solvable. Then the structure of a required optimum portfolio will correspond to a vector – $x = \{x_i\}$ $i = \overline{1,N}$ – the solution to the problem (12) - (16) or (17) - (21), the criterion function value of which will be greater.

V. The Comparison and Analysis of the Results Received Using Markowitz and Fuzzy Set Models

The experimental investigations of fuzzy portfolio optimization problem were carried out in [5].

To perform the comparative analysis of investigated methods of share portfolio optimization, real data on share prices of the companies RAO EES (EERS2) and Gazprom (GASP) were taken from February 2000 till May 2006.

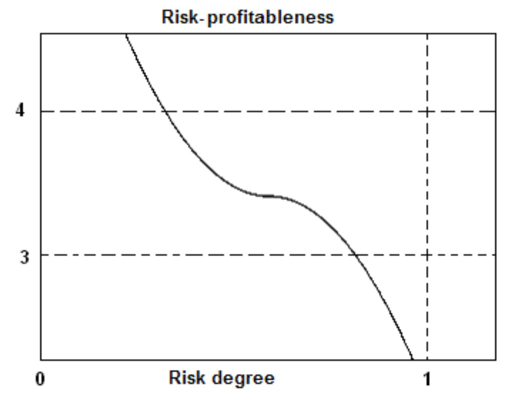
In the Markowitz model, the expected profitability of the share is calculated as a mean $m = M\{r\}$ and the risk of an asset is considered to be dispersion of the expected profitability value $\sigma^2 = M\left[(m - r)^2\right]$.

In the fuzzy set model proceeding from the situation on the share market:

- share profitability of EERS2 is in the settlement corridor [-1.0; 3.9], the most expected value of profitability is 2.1%.
- share profitability of GASP is in the settlement corridor [-4.1; 5.7], the most expected value of profitability is 4.8%.

Let the critical profitability of a portfolio be r^*=3.5%, i.e., portfolio investments, which bring the income below 3.5%, are considered inefficient.

The structures of an optimum portfolio, obtained as a result of the use of both methods, for the same risk levels are quite different. To find out the reason of this, we consider the following dependences (Fig. 1) [5].



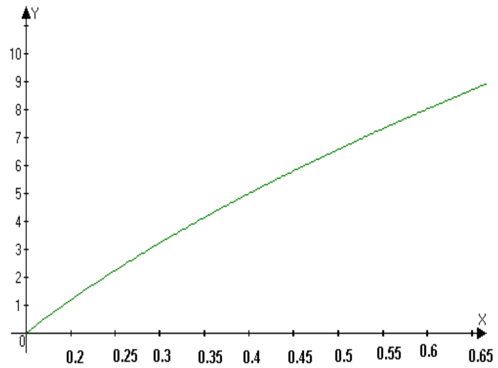


Fig. 1. a) using the fuzzy set method, b) using the Markowitz model

Dependencies of the expected profitability on portfolio risk degree, obtained by means of the specified methods, are practically opposite. The reason of such a result is the different understanding of a portfolio risk level.

In the fuzzy set method, the risk is recognized as a situation when the expected profitability of a portfolio drops below the critical level, with the decrease in the expected profitability the risk of the income of portfolio investments is less than a critical value [5, 10].

In the Markowitz model, the risk is considered to be the degree of expected portfolio income variability, to a lesser or greater extent, that contradicts common sense. The different understanding of portfolio risk level is also the reason of distinction of risk degree dependencies on share profitability in a portfolio, obtained by means of different methods.

The level of variability of the expected income for shares EERS2 based on the data 2000-2006 is much lower than for shares GASP. Therefore, in the Markowitz model, which considers it to be a risk of portfolio investments, with the increase in the ratio of share EERS2 the risk of a portfolio decreases.

From the point of view of the fuzzy set approach, the more the ratio of GASP shares in a portfolio, the less the risk is, so that the efficiency of share investments will appear to be below the critical level, which is 3.5% in our case.

VI. The Dual Fuzzy Portfolio Problem

The initial portfolio optimization problem, which is naturally to be called as direct, has the form (10-14) [5].

Let us consider the case when the criterion value r^* meets the conditions

$$\sum_{i=1}^{N} x_i r_{i1} \le r^* \le \sum_{i=1}^{N} x_i \tilde{r}_i = \tilde{r} \ . \qquad (22)$$

Then

$$\left(\left(r^* - \sum_{i=1}^{N} x_i r_{i1} \right) + \left(\sum_{i=1}^{N} x_i \tilde{r}_i - r^* \right) \cdot \ln \left(\frac{\sum_{i=1}^{N} x_i \tilde{r}_i - r^*}{\sum_{i=1}^{N} x_i \tilde{r}_i - \sum_{i=1}^{N} x_i r_{i1}} \right) \right) \cdot$$

$$\cdot \frac{1}{\sum_{i=1}^{N} x_i r_{i2} - \sum_{i=1}^{N} x_i r_{i1}} = \beta(x),$$

Now consider the dual portfolio optimization problem related to the problem (1)-(4).

To minimize $$\beta(x) \qquad (23)$$

under conditions $$\tilde{r} = \sum_{i=1}^{N} x_i \tilde{r}_i \ge r_{зад} = r^* \qquad (24)$$

$$\sum_{i=1}^{N} x_i = 1, \quad x_i \ge 0 \qquad (25)$$

In works [7, 11] the sufficient conditions were found under which the risk function $\beta(x)$ was convex. The sufficient conditions are the following [11]:

$$\sum_{i=1}^{N} x_i r_{i1} \le r^* \le \sum_{i=1}^{N} x_i \tilde{r}_i = \tilde{r}$$

In this case, the dual portfolio problem (23)-(25) is a convex programming problem. Taking into account that constraints (8) are linear, let us compose the Lagrangian function

$$L(x,\lambda,\mu) = \beta(x) + \lambda \left(r^* - \sum_{i=1}^{N} x_i \tilde{r}_i \right) + \mu \left(\sum_{i=1}^{N} x_i - 1 \right). \qquad (26)$$

The optimality conditions proposed by Kuhn-Tucker are the following:

$$\frac{\partial L}{\partial x_i} = \frac{\partial \beta(x)}{\partial x_i} - \lambda r_i + \mu \ge 0, \ i = \overline{1,N},$$
$$\frac{\partial L}{\partial \mu} = \sum_{i=1}^{N} x_i - 1 = 0, \qquad (27)$$

$$\frac{\partial L}{\partial \lambda} = -\sum_{i=1}^{N} x_i \tilde{r}_i + r^* \le 0. \qquad (28)$$

and conditions of complementary non-fixedness

$$\frac{\partial L}{\partial x_i} x_i = 0,\ i = \overline{1,N},$$
$$\frac{\partial L}{\partial \lambda}\lambda = \lambda\left(-\sum_{i=1}^{N} x_i \tilde{r}_i + r^*\right) = 0, \quad (29)$$
$$x_i \ge 0.$$

where $\lambda \ge 0$ and μ are the Lagrange multipliers.

This problem may be solved using standard methods of convex programming, for instance, using the method of feasible directions or penalty function method.

VII. Multicriteria Portfolio Optimization Problem

Now consider multicriteria fuzzy portfolio optimization problem, in which portfolio profitability should be maximized and risk should be minimized [11].

In order to find the structure of corresponding fuzzy portfolio, the following problem is to be solved:

$$\{x_{opt}\} = \{x\} \quad | \quad r \to \max, \quad \beta \to \min, \quad (30)$$

where r and β are determined by formulas (1), (13, (18) and vector X components satisfy (25).

For simplifying the problem solution, it is necessary to transform it to a single criterion. Normalize the value of profitability as follows:

$$\tilde{r}_н = \frac{r_{\max} - \tilde{r}}{r_{\max} - r_{\min}}, \quad \tilde{r}_н \in [0;1], \quad (31)$$

Using formulas (18), (31), we obtain the optimization problem in the following form:

$$\{w_1 \tilde{r}_н + w_2 \beta(x)\} \to \min$$
$$w_1 \ge 0,\ w_2 \ge 0,\ w_1 \ne w_2,\ w_1 + w_2 = 1$$
$$\sum_{i=1}^{N} x_i = 1,\ x_i \ge 0,\ i = \overline{1,N}$$

Consider $0 < \beta < 1$. It is possible in the two cases: if $\sum_{i=1}^{N} x_i r_{i1} \le r^* < \sum_{i=1}^{N} x_i \tilde{r}_i$ and if $\sum_{i=1}^{N} x_i \tilde{r}_i \le r^* < \sum_{i=1}^{N} x_i r_{i2}$.

a) Let it be $\sum_{i=1}^{N} x_i r_{i1} \le r^* < \sum_{i=1}^{N} x_i \tilde{r}_i$. Using (13) this problem is transferred as follows:

$$\{w_1 \tilde{r}_н + w_2 \beta(x)\} \to \min$$
$$w_1 \ge 0,\ w_2 \ge 0,\ w_1 \ne w_2,\ w_1 + w_2 = 1$$
$$\sum_{i=1}^{N} x_i r_{i1} \le r^*,$$
$$\sum_{i=1}^{N} x_i \tilde{r}_i > r^*,$$
$$\sum_{i=1}^{N} x_i = 1,\ x_i \ge 0,\ i = \overline{1,N}.$$

where β(x) is given by (13).

b) Let it be $\sum_{i=1}^{N} x_i \tilde{r}_i \le r^* < \sum_{i=1}^{N} x_i r_{i2}$, this problem is transferred as follows:

$$\{w_1 \tilde{r}_н + w_2 \beta(x)\} \to \min$$
$$w_1 \ge 0,\ w_2 \ge 0,\ w_1 \ne w_2,\ w_1 + w_2 = 1$$

where β(x) is given by (18).

$$\sum_{i=1}^{N} x_i r_{i2} > r^*,$$
$$\sum_{i=1}^{N} x_i \tilde{r}_i \le r^*,$$
$$\sum_{i=1}^{N} x_i = 1,\ x_i \ge 0,\ i = \overline{1,N}.$$

VIII. Experimental Investigations and Result Analysis

As the input data, the closing prices of the leading companies at the stock exchange NYSE were used: Canon Inc. (ADR) (CAJ), Hewlett-Packard Company (HPQ), McDonald's Corporation (MCD), Microsoft Corporation (MSFT), PepsiCo, Inc (PEP), The Procter & Gamble Company (PG), SAP AG (ADR) (SAP) in the period from 5 December 2011 to 30 March 2012. The corresponding data are given in Table I.

TABLE I
Closing Prices

Company / Date	CAJ	HPQ	MCD	MSFT	PEP	PG	SAP
5 December 2011	44.2	28.12	95.35	25.7	64.4	64.84	58.38
9 December 2011	44.76	27.90	98.03	25.7	65.19	64.97	58.78
12 December 2011	44.39	27.34	98.48	25.51	64.66	64.31	57.29
16 December 2011	43.02	25.84	97.49	26	64.71	65.14	54.61
19 December 2011	43.27	25.13	97.24	25.53	64.37	64.95	54.31
23 December 2011	44.47	25.88	100.15	26.01	66.57	66.67	53.25

27 December 2011	43.96	25.65	100.55	26.01	66.38	66.79	53.29
30 December 2011	44.04	25.76	100.33	25.96	66.35	66.71	52.95
3 January 2012	45.08	26.62	98.84	26.76	66.4	66.83	54.43
6 January 2012	43.55	26.40	100.6	28.01	65.39	66.36	54
9 January 2012	43.55	26.44	99.64	27.74	65.73	66.64	54.75
13 January 2012	43.56	26.49	100.35	28.25	64.4	65.81	54.56
17 January 2012	43.08	26.46	100.55	28.26	64.65	66.26	55.67
20 January 2012	44.5	28.13	101.74	29.71	66.28	66.23	57.01
23 January 2012	44.31	28.68	100.95	29.73	66.1	65	57.76
27 January 2012	45.14	27.88	98.69	29.23	65.81	64.3	60.35
30 January 2012	43.7	27.88	98.69	29.61	65.41	63.21	60.35
3 February 2012	44.21	29.07	100.01	30.24	66.66	62.77	63.17
6 February 2012	44.23	28.76	99.49	30.2	66.52	63.51	62.99
10 February 2012	43.69	28.70	99.47	30.5	63.95	63.88	62.9
13 February 2012	44.32	28.75	99.65	30.58	63.69	64.23	63.33
17 February 2012	45	29.59	99.99	31.25	62.68	64.91	64.44
21 February 2012	45.16	29.35	100.49	31.44	63.14	64.42	65.37
24 February 2012	45.2	26.64	100.32	31.48	63.31	66.71	67.77
27 February 2012	45.46	26.25	100.36	31.35	63.32	66.7	67.01
2 March 2012	45.65	25.32	99.5	32.08	62.52	66.67	67.66
5 March 2012	45.52	25.01	99.94	31.8	62.79	66.95	68.1
9 March 2012	46.12	24.18	96.84	31.99	63.15	66.93	68.99
12 March 2012	45.2	24.04	96.66	32.04	63.94	67.71	69.39
16 March 2012	47.32	24.49	97.66	32.6	64.47	67.25	72
19 March 2012	46.97	24.34	97.73	32.2	64.73	67.21	72.31
23 March 2012	46.74	23.63	95.55	32.01	65.3	67.43	70.38
26 March 2012	47.36	23.89	96.97	32.59	65.78	67.46	71.24
30 March 2012	47.66	23.83	98.1	32.26	66.35	67.21	69.82

The corresponding profitability is presented in Table II.

TABLE II

PROFITABILITY, %

Company / Date	CAJ	HPQ	MCD	MSFT	PEP	PG	SAP
9 December 2011	1.251	-0.789	2.734	0.000	1.212	0.200	0.681
16 December 2011	-3.185	-5.805	-1.015	1.885	0.077	1.274	-4.908
23 December 2011	2.698	2.898	2.906	1.883	3.305	2.580	-1.991
30 December 2011	0.182	0.427	-0.219	-0.308	-0.045	-0.120	-0.642
6 January 2012	-3.513	-0.833	1.750	4.769	-1.545	-0.708	-0.796
13 January 2012	0.023	0.189	0.708	1.805	-2.065	-1.261	-0.348
20 January 2012	3.191	5.937	1.170	4.881	2.459	-0.045	2.350
27 January 2012	1.839	-2.869	-2.290	-1.711	-0.441	-1.089	4.292
3 February 2012	1.154	4.094	1.320	2.083	1.875	-0.701	4.464
10 February 2012	-1.236	-0.209	-0.020	0.984	-4.019	0.579	-0.143
17 February 2012	1.511	2.839	0.340	2.144	-1.611	1.048	1.723
24 February 2012	0.088	-10.173	-0.169	0.127	0.269	3.433	3.541
2 March 2012	0.416	-3.673	-0.864	2.276	-1.280	-0.045	0.961
9 March 2012	1.301	-3.433	-3.201	0.594	0.570	-0.030	1.290
16 March 2012	4.480	1.837	1.024	1.718	0.822	-0.684	3.625
23 February 2012	-0.492	-3.005	-2.282	-0.594	0.873	0.326	-2.742

Further using the Fuzzy GMDH method [10] with triangular membership functions, linear partial descriptions and training sample of 70%, the next profitability values were predicted by 30 March 2012 (Table III):

TABLE III
PREDICTED VALUES OF SHARE PROFITABILITY, %

Company	Profitability				MAPE test sample	MSE test sample
	Real	Low bound	Predicted	Upper bound		
CAJ	0.629	0.476	0.646	0.816	2.5877	0.0163
HPQ	-0.252	-0.372	-0.242	-0.112	3.9231	0.0099
MCD	1.152	0.915	1.184	1.454	2.8177	0.0325
MSFT	-1.023	-1.275	-1.005	-0.735	1.7525	0.0179
PEP	0.859	0.725	0.836	0.946	2.7129	0.0233
PG	-0.372	-0.470	-0.381	-0.292	2.3729	0.0088
SAP	-2.034	-2.219	-1.997	-1.776	1.7877	0.0364

Thus, as a result of application of FGMDH, the shares profitability values were predicted by the end of 13th week (30 March 2012):

e.g. profitability of CAJ shares lies in the calculated corridor [0.476; 0.816], the expected value is 0.646%;

• And so forth.

Thus, the portfolio optimization system stops to be dependent on the factor of expert's subjectivity. Besides, we can obtain data for this method automatically, without expert's estimates.

Let us consider the results of application of the suggested approach to the determining of optimal invest portfolio by 30 March 2012.

Let the critical profitability level set by a trader be 0.9%. By varying the risk level, we obtain the following results for triangular MF presented in Table IV, V and Fig. 2.

TABLE IV
COMPONENTS OF OPTIMAL PORTFOLIO FOR TRIANGULAR MF WITH CRITICAL LEVEL r^*=0.9%

CAJ	HPQ	MCD	MSFT	PEP	PG	SAP
0.00606	**0.00257**	**0.91245**	**0.00218**	**0.00972**	**0.00251**	**0.06451**
0.00931	0.00467	0.89796	0.0061	0.00938	0.00347	0.06911
0.00392	0.00587	0.88673	0.01875	0.00608	0.00686	0.0718
0.01374	0.00449	0.86448	0.02416	0.01838	0.00161	0.07314
0.00811	0.00376	0.88901	0.00617	0.01088	0.00283	0.07925
0.0056	0.00558	0.87824	0.01136	0.00494	0.00395	0.09033
0.00962	0.00463	0.86839	0.04609	0.00788	0.00305	0.06034
0.00636	0.00606	0.8437	0.07917	0.00307	0.00844	0.0532

TABLE V
PARAMETERS OF OPTIMAL PORTFOLIO WITH CRITICAL LEVEL r^*=0.9%

Low bound	Expected profitability	Upper bound	Risk level
0.69677	**0.9598**	1.22366	**0.35**
0.66838	0.93046	1.19336	0.4
0.629	0.89125	1.1543	0.45
0.61535	0.87575	1.13693	0.5
0.63875	0.90048	1.163	0.55
0.59099	0.85293	1.11566	0.6
0.60906	0.87189	1.13551	0.65
0.55202	0.81512	1.07901	0.7

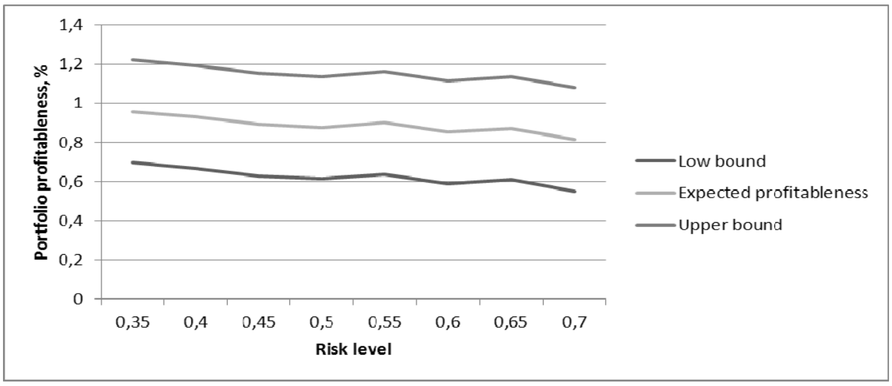


Fig. 2. Dependence of the expected portfolio profitability on the risk level for triangular MF

As seen in Fig. 2, the dependence has a descending type; the greater the risk the lesser profitability is opposite to classical probabilistic methods. It may be explained so that at fuzzy approach by risk is meant the situation when the expected profitableness happens to be less than the given criteria level. When the expected profitability decreases, the risk grows. The profitability of a real portfolio equals 0.9601%. This value falls in the found calculated corridor of profitability [0.69677; 0.9598; 1.22366] that proves the high forecast accuracy.

The profitability of a real portfolio is equal to 1.13304%. This value falls in the calculated corridor of profitability [0.882312; 1.143272; 1.405132].

Now consider the same portfolio with the Gaussian membership function (MF) (see Table VI, VII and Fig. 3)

TABLE VI

OPTIMAL PORTFOLIO COMPONENTS OBTAINED USING THE GAUSSIAN MF AND CRITERIA LEVEL OF 0.9%

CAJ	HPQ	MCD	MSFT	PEP	PG	SAP
0.01426	0.00201	0.94619	0.00103	0.03327	0.00103	0.00222
0.01403	0.00286	0.94533	0.00119	0.03335	0.00097	0.00228
0.01572	0.00099	0.94279	0.00111	0.03507	0.00099	0.00333
0.01883	0.00255	0.93679	0.00106	0.03611	0.00124	0.00342
0.01861	0.00185	0.9345	0.00381	0.03676	0.0015	0.00298
0.02059	0.00161	0.93407	0.001	0.03791	0.00104	0.00379
0.02044	0.00101	0.93473	0.00093	0.03869	0.00102	0.00318
0.02104	0.00187	0.92949	0.00359	0.0393	0.00098	0.00374

TABLE VII

OPTIMAL PORTFOLIO PARAMETERS OBTAINED USING THE GAUSSIAN MF AND CRITERIA LEVEL OF 0.9%

Low bound	Expected profitability	Upper bound	Risk level
0.8892	**1.15097**	1.41364	**0.35**
0.86772	1.12939	1.39196	0.4
0.848306	1.109736	1.372076	0.45
0.848453	1.1091425	1.370733	0.5
0.860248	1.1209284	1.382508	0.55
0.829026	1.0894159	1.350696	0.6
0.831054	1.0914435	1.352724	0.65
0.801251	1.0613507	1.322331	0.7

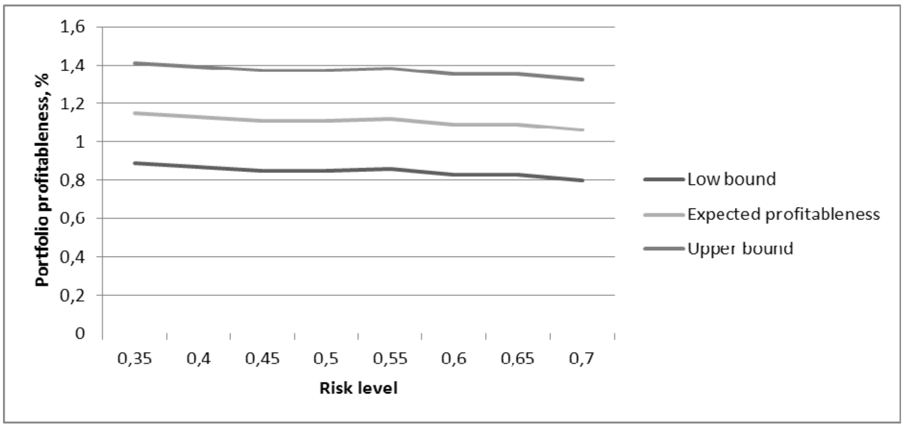


Fig. 3. The dependence of expected portfolio profitability on the risk level

Profitability of a real portfolio is equal to 1.15128%. This value drops to the found calculated corridor for profitability [0.8892; 1.15097; 1.41364].

Optimal portfolios obtained with the application of different MF have the same structure. The main portfolio fraction belongs to company MCD that can be explained by a high level of its profitability in comparison with other companies.

Now consider the results, obtained while solving dual portfolio problem by means of the triangular MF. In this case, an investor sets the profitability level and the problem is to minimize the risk. The optimal portfolio built of seven components is presented in Table VIII, IX.

TABLE VIII

OPTIMAL PORTFOLIO COMPONENTS (DUAL PROBLEM)

CAJ	HPQ	MCD	MSFT	PEP	PG	SAP
0.02145	0.01319	0.92188	0.00544	0.0235	0.01204	0.0025
0.02392	0.01223	0.92175	0.00154	0.02672	0.01056	0.00329
0.02534	0.00895	0.92689	0.00205	0.02911	0.00649	0.00116
0.02659	0.00598	0.9298	0.00366	0.03119	0.00286	0.00008
0.02711	0.00207	0.93261	0.00151	0.03253	0.00113	0.00304
0.02606	0.00101	0.93639	0.00142	0.03295	0.0012	0.00098
0.02684	0.0003	0.93673	0.00181	0.03202	0.00025	0.00206

TABLE IX

OPTIMAL PORTFOLIO PARAMETERS (DUAL PROBLEM)

Low bound	Expected profitability	Upper bound	Risk level
0.84773	1.10678	1.36673	0.0092
0.85539	1.11405	1.3736	0.03096
0.8697	1.12913	1.38945	0.0666
0.87797	1.13807	1.39907	0.12765
0.87985	1.1404	1.40184	024107
0.88817	1.14898	1.4107	0.41426
0.88599	1.1471	1.4091	0.7926

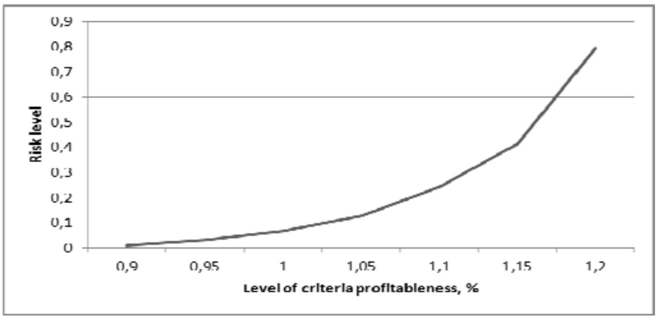


Fig. 4. The dependence of the risk level on the criteria profitability for a dual problem

Consider the results of the application of multicriteria problem to the optimal portfolio construction In this case, an investor can set his own preference to profitability or risk, adapting weight coefficients w_1 and w_2 correspondingly. Thus, the optimal portfolio was obtained under the set level of profitability criteria, which is presented in Table X, XI and Fig. 5.

TABLE X
DISTRIBUTION OF OPTIMAL PORTFOLIO COMPONENTS WITH THE TRIANGULAR MF AND PROFITABILITY CRITERIA OF 0.9% (MULTICRITERIA PROBLEM)

CAJ	HPQ	MCD	MSFT	PEP	PG	SAP
0.02108	0.01339	0.92124	0.0062	0.02298	0.01232	0.00279
0.02166	0.01462	0.91832	0.00805	0.02339	0.01364	0.00033
0.02123	0.01485	0.91636	0.0089	0.0228	0.01396	0.0019
0.02076	0.0151	0.91427	0.00981	0.02216	0.0143	0.00359
0.01928	0.01435	0.91812	0.00976	0.02049	0.01366	0.00435
0.01874	0.01463	0.91571	0.01082	0.01974	0.01405	0.00631
0.01815	0.01494	0.91312	0.01196	0.01893	0.01448	0.00843
0.0175	0.01526	0.91032	0.01319	0.01804	0.01495	0.01074
0.01579	0.01461	0.91332	0.01352	0.01608	0.01445	0.01223

TABLE XI
OPTIMAL PORTFOLIO PARAMETERS OBTAINED WITH THE TRIANGULAR MF WITH THE APPLICATION OF CRITERIA LEVEL OF 0.9% (MULTICRITERIA PROBLEM)

Low bound	Expected profitability	Upper bound	Risk level	w_1
0.84476	**1.10384**	1.36382	0.01043	**0.1**
0.8447	1.10336	1.36292	0.00925	0.2
0.83745	1.09609	1.35562	0.01184	0.3
0.82968	1.0883	1.3478	0.01501	0.4
0.83026	1.08946	1.34956	0.01765	0.5
0.82126	1.08043	1.3405	0.02223	0.6
0.81153	1.07068	1.33071	0.02784	0.7
0.801	1.06011	1.32011	0.03474	0.8
0.79825	1.05796	1.31854	0.0416	0.9

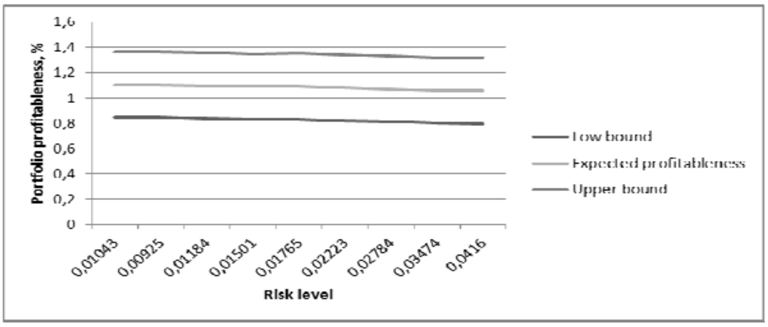


Fig. 5. The dependence of the expected profitability on the risk level

The profitability of a real portfolio is 1.1078%. This value drops to the calculated corridor of profitability [0.84476; 1.10384; 1.36382] that proves again the high forecast accuracy by means of the fuzzy GMDH method.

As one may see from the presented tables and Fig. 5, the dependence of profitability on the risk is also descending. When coefficient w_2 decreases, the risk level goes up. Thus, one may take into account the investor's preferences for risk and profitability using corresponding weight coefficients while constructing an optimal portfolio.

IX. Conclusions

In this paper, the research in the field of portfolio management has been carried out. Markowitz model, as the widely used method in the given area, and the fuzzy set approach recently suggested to portfolio optimization have been considered and compared. As a result of research, the mathematical model based on the fuzzy sets approach to find the structure of the optimum investment portfolio has been obtained, deprived of drawbacks of classical probabilistic models. On the basis of the fuzzy set theory, the algorithm of portfolio optimization has been developed. In the course of research and the comparative analysis of Markowitz model and fuzzy set model, the following conclusions have been drawn:

1. Structures of the optimal portfolio and the indicators of its expected profitability obtained by means of the Markowitz model and fuzzy set model principally differ.
2. The dependencies of the expected profitability on the risk level of portfolio obtained using the Markowitz model and fuzzy set model are completely opposite.
3. The dual portfolio optimization problem has been investigated, and the sufficient conditions of risk function convexity have been found.
4. Multicriteria portfolio optimization problem has been stated and investigated.
5. In order to improve the accuracy of the suggested fuzzy portfolio model, the fuzzy GMDH method has been applied to forecast the profitability. The experimental investigations have proved its high efficiency.

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Jurijs Zajčenko, Inna Sidoruka. Izplūduša investīciju portfeļa optimizācija nenoteiktības apstākļos
Rakstā tiek veikta klasiskā G. Markovica investīciju portfeļa optimizācijas modeļa analīze, izklāstīti šā modeļa galvenie trūkumi. Kā alternatīva klasiskajam modelim tiek aprakstīta vērtspapīru portfeļa optimizācijas uzdevumu nostādne, kas balstās uz izplūdušas kopas pieeju. Piedāvātajā uzdevumā akciju ienesīgums tiek definēts kā izplūduši skaitļi ar trijstūra vai Gausa piederības funkciju. Riska portfelis tiek aplūkots kā situācija, kad reālais portfeļa ienesīgums atrodas zem investora definētas zināmas kritērija vērtības. Uzbūvēts matemātiskais modelis un aprakstīts tā risinājuma algoritms izplūduša portfeļa optimizācijas uzdevumā. Veikti izplūduša modeļa eksperimentāli pētījumi, kas balstīti uz akciju portfeļa izveidošanu no kompānijām Krievijas biržā (Krievijas Tirdzniecības Sistēma). Veikti salīdzinošie eksperimentālie pētījumi ar klasisko Markovica un izplūdušas kopas modeļiem. Izveidotas sakarības starp abiem modeļu veidiem - optimālo ienesīgumu un risku, kā arī norādīts, ka atšķirībā no klasiskā modeļa dotā atkarība izplūdušā modelī ir monotoni nogalinoša. Tiek sniegts šīs izpausmes pamatojums. Izskatīts divējādi risināms izplūduša portfeļa optimizācijas uzdevums ar ierobežojumu, kurā nepieciešams minimizēt vērtspapīru portfeļa risku sagaidāmajam ienesīgumam. Noformulēts daudzkritēriju izplūduša portfeļa optimizācijas uzdevums ar diviem kritērijiem: portfeļa ienesīgums un risks. Atspoguļoti šī uzdevuma eksperimentālie pētījumi. Šī uzdevuma ietvaros galvenais nenoteiktības avots ir akciju cenas, kuru vērtības zināmas portfeļa izveidošanas brīdī, bet nav zināmas nākotnē, realizējot šo portfeli. Lai samazinātu nenoteiktību un pazeminātu risku, tiek piedāvāts izmantot akciju ienesīguma prognozēšanu. Prognozēšanai tiek piedāvāts izmantot speciālu induktīvo modelēšanu - izplūdušas grupas datu apstrādes metodi (GDAM), kura nodrošina iespēju apstrādāt ieejas datu nenoteiktību un automātiski uzbūvēt nepieciešamo prognozes modeli, balstoties uz eksperimentāliem datiem. Eksperimenti pierādīja izplūdušas GDAM izmantošanas efektivitāti akciju cenu prognozēšanai nenoteiktības apstākļos investīciju portfeļa optimizācijas uzdevumā.

Юрий Зайченко, Инна Сидорук. Оптимизация нечёткого инвестиционного портфеля в условиях неопределённости
В статье дан анализ классической модели оптимизации инвестиционного портфеля Г. Марковица, изложены ее основные недостатки. В качестве альтернативы классической модели описана постановка задачи оптимизации портфеля ценных бумаг на основе нечётко-множественного подхода. В этой задаче доходности акций описываются как нечёткие числа с треугольной или Гауссовской функцией принадлежности. При этом риск портфеля рассматривается как ситуация, когда реальная доходность портфеля оказывается ниже некоторого критериального значения, задаваемого инвестором. Построена математическая модель задачи оптимизации нечеткого портфеля и описан алгоритм ее решения. Проведены экспериментальные исследования нечёткой модели на примере составления портфеля из акций компаний на Российской бирже РТС (Российская Торговая Система). Проведены сравнительные экспериментальные исследования классической модели Марковица и нечётко-множественной модели. Построены зависимости «оптимальная доходность – риск» для обоих типов моделей и показано, что в отличие от классической модели данная зависимость для нечёткой модели является монотонно убывающей. Дано обоснование этого явления. Рассмотрена двойственная задача нечеткой портфельной оптимизации, в которой необходимо минимизировать риск при ограничении на ожидаемую доходность портфеля. Сформулирована многокритериальная задача нечёткой портфельной оптимизации по двум критериям: доходности портфеля и риску, проведены её экспериментальные исследования. В данной задаче основным источником неопределённости являются цены акций, значения которых известны в текущий момент конструирования портфеля и неизвестны в будущий момент реализации этого портфеля. С целью уменьшения неопределённости и снижения риска предложено использовать прогнозирование доходности акций. Для прогнозирования предлагается использовать специальный метод индуктивного моделирования - нечёткий МГУА, позволяющий учесть неопределённость исходных данных и автоматически построить искомую модель прогноза по экспериментальным данным. Эксперименты подтвердили эффективность применения нечёткого МГУА для прогнозирования цен акций в задаче оптимизации инвестиционного портфеля в условиях неопределённости.