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Probability Weighting in Decision-Making Tasks under Risk

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Abstract **– The analysis of alternative decisions and the choice of the optimal – in a given sense – decision is an integral part of people's purposeful activity in all areas of their social life. Many formal approaches have been proposed to solve these problems. One such approach is expected utility theory, which correctly models individuals' subjective preferences and attitudes to risk. For a very long time this theory was the leading approach for decision making under conditions of risk. However, numerous practical studies have shown its weakness: the theory did not explicitly use subjective perceptions of decision outcome probabilities in optimal decision-making processes. This research has led to the creation and development of approaches to explicitly consider the probabilities of outcomes in decision making. This paper provides a critical analysis of the descriptive properties of expected utility theory and presents various forms of probability weighting functions.**

Keywords – **Allais paradox, decision making under risk, expected utility, probability weighting.**

I. Introduction

The origins of probability theory lie in gambling. The widespread use of such games in the seventeenth century required calculations of the players' odds in various gambling situations. The emergence of probability theory as a science of chance was a response to the peculiar needs of society.

Even before the appearance of probability theory, it was clear that the goal of any player was to win as much as possible. When it became clear that the players' chances of winning depended on certain random events (favourable cards, a certain number of dice rolls), the expected winnings became a measure of the players' successes and failures. This winning could be calculated by multiplying the probabilities of different game outcomes by the corresponding winnings and losses and summing up the obtained results. At the same time, a fundamental principle of gambling was formulated: to maximise the players' expected winnings. Nowadays this principle, in the context of decision making, is formulated as maximising the expected value over the whole set of alternative solutions.

The concept of maximising expected winnings was transparent and understandable to professional players. However, as knowledge and experience accumulated, it became clear that, in specific circumstances, the concept went against simple reasoning logic. Let us consider a simple example. Let it be known that due to a slight asymmetry a coin goes tails in 52 cases out of 100. The player is offered the following game: he bets in conventional monetary units (cmu). Then a coin is tossed. If the coin goes tails, he wins x cmu. If the coin goes heads, he loses his bet. The expected winnings in this game are calculated very simply

$$E = 0.52x - 0.48x = 0.04x\,.$$

Since the expected winnings are positive, the player may be advised to take part in this game. However, obviously, only a very risky person would take part in such a game. Most sensible people will refuse to play, since the odds of winning and losing are only slightly different from each other.

How to solve this paradox? The first answer to this question was given by D. Bernoulli in 1738. He put forward the ingenious idea that the utility of money does not increase in direct proportion to its quantity, but in a more complex way, namely as the logarithm of the quantity of money. Modern evidence shows that Bernoulli's assumption of a logarithmic relationship between the utility of money and the quantity of money only takes place in certain specific situations.

Unfortunately, D. Bernoulli's fruitful ideas about utility have been forgotten for almost 200 years. However, it does not follow that the concept of utility itself has been forgotten. The concept has been used extensively in economics, the study of consumer demand and other areas of human activity. An empirical study of the behaviour of individuals in risky choice situations (gambling, lotteries, etc.) has shown that rationally thinking individuals in such situations act to maximise their utility. In other words, the concept of utility existed, but there was no theoretical justification for the concept.

A step in the direction of developing a grounded theory of utility was the work of E. P. Ramsey and the work of B. de Finetti. These works are related to the theoretical underpinning of subjective probability theory. Both works assumed that when individuals estimate subjective probabilities on the basis of a wager, they act in such a way as to maximise their expected utility. Thus, in these papers a bridge was built between utilities and probabilities.

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II. Conceptual Foundations of Expected Utility Theory

The first paper that successfully laid the theoretical foundations for expected utility was [1]. In this work, the authors proposed a system of axioms about the individual's preferences on a set of risky lotteries (games). They proved that if the requirements of these axioms are satisfied, an individual's utility function can be constructed, and the best actions of an individual in risky choice situations are those that maximize expected utility.

An essential feature of the von Neumann and Morgenstern approach was that the probabilities of relevant outcomes of lotteries (games) were assumed to be known and determined in an objective way.

In [2], the author proposed a different axiomatic basis for expected utility. In essence, Savage's approach is a synthesis of the ideas of de Finetti and von Neumann and Morgenstern. His results consist in both a complete theory of subjective probabilities and a complete theory of expected utility. Subjective probabilities are estimated based on the principle of maximising expected utility. In turn, the subjective probability estimates are used to estimate the utility of risky actions.

Given the fundamental difference between von Neumann and Morgenstern's and Savage's theories regarding a priori existence of probability estimates, von Neumann and Morgenstern's theory is called the expected utility theory under risk, while Savage's theory is called the expected utility theory under uncertainty.

Expected utility theory has found wide application in problems of optimal decision choice under risk. Suppose there is a standard decision choice problem under risk. Assume that all factors of the problem are known and the decision situation is modelled appropriately, e.g., using a decision tree. The first step in solving this problem is to construct a decision maker's utility function $u(K)$ on the set of values of the evaluation criterion K. Then, using the constructed utility function $u(K)$ the criterion estimates of all outcomes are transformed into corresponding utility estimates. The next step is to calculate the expected utility for each alternative decision a_j, $j = 1,\ldots,m$.

$$u\left(a_j\right) = \sum_{i=1}^{n} u\left(k_{ji}\right) p_{ji} , \tag{1}$$

where $u(k_{ij})$ – utility estimate of the i-th outcome of the j-th alternative decision;
p_{ij} – probability of occurrence of the i-th outcome of the j- th alternative decision.

The optimum decision is chosen according to the condition

$$\operatorname{opt}\left(a_j\right) \to \max_j u\left(a_j\right) \to \max_j \sum_{i=1}^{n} u\left(k_{ji}\right) p_{ji} . \tag{2}$$

Expected utility theory is an axiomatic theory. One of the axioms is the axiom of independence. Let there be three alternatives (games, lotteries, solutions) A, B, C, $A \succeq B$ and $\alpha \in (0,1]$. Then

$$\alpha A + \left(1-\alpha\right) C \succeq \alpha B + \left(1-\alpha\right) C . \tag{3}$$

This axiom states that an individual has well-defined preferences on sets of perspectives. If the individual prefers perspective A to perspective B, then according to (3) he must prefer a linear combination (mixture) of perspectives A and C to a linear combination (mixture) of perspectives B and C.

We have given here the independence axiom of the expected utility theory because it will play an important role in the subsequent analysis of the descriptive properties of this theory. Let us call the fact that expected utility satisfies the independence axiom, the first characteristic feature of the theory.

The second characteristic of expected utility theory is that the form of the utility function simultaneously reflects an individual's preferences for a set of decision outcomes and his attitude towards risk. Let us define this concept in more detail. Suppose we have a decision

$$A = \left(u\left(x_1\right), p_1; \ldots, u\left(x_n\right), p_n\right),$$

where $u(x_i)$, $i = 1,\ldots,n$, – utility estimate of the i-th outcome, p_i, $i = 1,\ldots,n$ – probability of occurrence of the i-th outcome, and the outcomes are ordered so that $u(x_1)<u(x_2)<\ldots<u(x_n)$.

Let us decrease the value of the probability of some outcome of the decision A by a value δ and add this value to the probability of the outcome with a larger value of expected utility. The result is a new solution A'. If we calculate the expected utility values of $U(A)$ and $U\left(A'\right)$, we have the following obvious relation $U\left(A'\right) < U(A)$. This is due to the transfer of a certain fraction of the probability δ from one outcome to the other. This transfer decreases the expected utility of the first outcome and increases the expected utility of the other outcome. Obviously, the increase in the expected utility of the second outcome is greater than the decrease in the expected utility of the first outcome. It follows that $U\left(A'\right) < U(A)$ and $A' \succ A$.

Thus, the notion of stochastic perspective dominance means that shifting some proportion of probability from a less favourable outcome of a prospect to a more favourable one results in a more favourable decision.

The fourth characteristic of the expected utility theory is as follows. To calculate the expected utility of a decision, only estimates of the utility of its outcomes and values of the probabilities of these outcomes are needed. At the same time, the values of the probabilities of outcomes are included in the calculation of the expected utility of a decision as coefficients, i.e., the value of the expected utility depends linearly on the values of the relevant probabilities. An individual's preferences on the set of outcomes and his attitude to risk are modelled using the utility function alone. Thus, an individual's preferences are not directly related to the values of the probabilities of the outcomes; these values only contribute to the prospect's estimate of expected utility. This conclusion follows directly from the formal definition of expected utility.

The four properties discussed above characterise expected utility theory as a purely normative theory of decision-making.

The theory has been shown to work in many practical applications. Economists often view this theory as descriptive in the sense that it describes the rational behaviour of individuals. From the perspective of decision choice, the theory is seen as a normative theory that specifies how individuals should act in risky choice situations. In [3], it is argued that expected utility theory can also be seen as a predictive or positivist theory. This is because the theory exhibits high predictive accuracy compared to other competing approaches.

III. Criticism of the Descriptive Properties of Expected Utility Theory

Experiments to test the descriptive properties of expected utility theory began soon after its emergence. One of the first such experiments was organised by W. Edwards [4], [5]. The aim of the experiments was to check whether individuals in risky choices really behaved as if they were maximising the expected utility of relevant prospects, or whether they were also taking into account in their choices the probabilities of the outcomes of the evaluated prospects. The results of the experiments showed unambiguously that in their choices, individuals focused not only on the gains and losses of the prospects, but also on their probabilities.

Regarding these results, the author rightly notes "People prefer relatively low loss probabilities (and relatively high loss quantities) and avoid relatively high loss probabilities and relatively low loss quantities. These findings are compatible with the hypothesis that people view events with negative expected values as less plausible than the same events with positive expected values, although they do not prove this hypothesis."

Similar results are also presented in [6]. In [7] the author, using a witty example of deterministic lottery outcomes, showed that individuals in risky choice situations implicitly consider the probabilities of the outcomes of alternative decisions.

The descriptive properties of expected utility theory have been hit hard by the so-called Allais paradox [8]. This paradox represents specially constructed choice situations that are offered to individuals. From the results of the choices we can confidently assess whether they are made on the basis of the expected utility theory or on another basis.

Individuals are asked to choose their preferred lottery in the following two choice situations.

Choice 1

Lottery 1:

$$A = (5000000, 0; 1000000, 1; 0, 0) \text{ or}$$

$$B = (5000000, 0.10; 1000000, 0.89; 0, 0.01).$$

Choice 2

Lottery 2:

$$A = (5000000, 0; 1000000, 0.11; 0, 0.89) \text{ or}$$

$$B = (5000000, 0; 1000000, 0.11; 0, 0.89).$$

In these lotteries, winnings are expressed in terms of cmu. For the sake of brevity, we will omit these notations in the following statement.

The lottery 1*A* has a guaranteed winning probability of 1 000 000 and a zero chance of winning 5 000 000 and 0. In the lottery 1*B*, there is a 0.10 chance of winning 5 000 000, a 0.89 chance of winning 1 000 000 and a zero chance of not winning anything.

In the lottery 2*A*, there is a 0.11 probability of winning 1 000 000, no chance of winning 5 000 000 and a 0.89 probability of not winning anything. In the lottery 2*B*, there is a probability of 0.10 to win 5 000 000, no chance of winning 1 000 000 and a probability of 0.89 to win nothing.

According to expected utility theory, the lottery 1*A* should be preferred to the lottery 1*B* and the lottery 2*A* should be preferred to the lottery 2*B*, or the lottery 1*B* should be preferred to the lottery 1*A* and the lottery 2*B* should be preferred to the lottery 2*A*.

When a group of individuals was asked to choose their preferred lotteries in choice situations 1 and 2, the actual selection results were as follows: in the first choice situation most individuals chose lottery 1*A*; in the second choice situation, most individuals chose lottery 2*A*. This choice is in complete contradiction to the foundations of expected utility theory.

In their actual choices, individuals were not guided by the maximisation of expected utility, but by some other criteria. On a simple worldly level, their choices can be justified as follows. Their choice of the lottery 1*A* was associated with an extreme reluctance to take a risk, because the lottery is not really a lottery in the conventional sense, since it is a guaranteed prize of 1 000 000. Here, the individuals were acting on the principle that "a bird in the hand is better than a crane in the sky". In the lottery 2*A* with a probability of 0.10 you can win 5 000 000; in the lottery 2*B* with a probability of 0.11 you can win 1 000 000. Since the odds of winning in the two lotteries differ slightly, but the winnings differ significantly, choosing the lottery can be justified as applying the principle "if you take a risk, take a big risk".

The Allais paradox is associated with two effects: *the general ratio effect and the common consequence effect*. (These effects are sometimes called paradoxes in the literature because they are related to the Allais paradox). The general ratio effect can be clearly demonstrated with the following example of the Allais paradox type [9]. There are two choices between lotteries.

Choice 1

Lottery *A*: reliably get 3000;

Lottery *B*: (4000, 0.80; 0, 0.20).

Choice 2

Lottery *A*: (3000; 0.25; 0, 0.75);

Lottery *B*: (4000, 0.20; 0, 0.80).

Most individuals prefer the lottery 1*A* to the lottery 1*B* and the lottery 2*B* to the lottery 1*A*. However, according to expected utility theory, individuals should have the following preferences: 1*A* preferred to 1*B* and 2*A* preferred to 2*B*, or 1*B* preferred to 1*A* and 2*B* preferred to 2*A*.

Let us show that the choices of most individuals do indeed contradict the theory of expected utility. If we construct a utility function, the choice $1A \succ 1B$ concludes that

$$u(3000) > 0.8u(4000).$$

On the other hand, the choice $2A \prec 2B$ concludes that

$$0.25u(3000) < 0.20u(4000).$$

Multiplying both parts of the last inequality by 4 gives

$$u(3000) < 0.8u(4000).$$

This result is in complete contradiction to the result obtained on the basis of the first choice.

In expected utility theory, the principle of ratio independence applies, which states that the results of an election are independent of the general ratio α. Formally, the principle can be expressed as

$$1A = (x, p; 0, 1-p) \succ 1B = (y, q; 0, 1-q),$$

if and only if

$$2A = (x, \alpha p; 0, 1-\alpha p) \succ 2B = (y, \alpha q; 0, 1-\alpha q).$$

In the lottery presented above, $1A$, $p = 1$ and in the lottery $1B$, $q = 0.80$. Let us multiply the values of p and q by 0.25. We have $\alpha p = 0.25$, $\alpha q = 0.20$. It follows that the probabilities of the lotteries $1A$ and $1B$ are related with the probabilities of the lotteries $2A$ and $2B$, by the relation, namely, the former are obtained by multiplying the latter with 0.25.

The actual choice of individuals on the set of lotteries presented above explicitly violates the principle of attitude independence and is therefore inadmissible from the point of view of expected utility theory.

In a more general formulation, the effect of the general ratio can be represented by the following two pairs of lotteries:

$$1C = (X, p; 0, 1-p); \quad 1D = (Y, q; 0, 1-q).$$

$$2C = (X, \alpha p; 0, 1-\alpha p); \quad 2D = (Y, \alpha q; 0, 1-\alpha q),$$

where $\alpha \in (0,1)$.

The second effect – the *common consequence effect* – can be demonstrated with the following illustrative example [9]. Individuals are asked to choose their preferred lottery in each of the following pairs of lotteries.

Choice 1

Lottery *A*: a guaranteed income of 500 000;

Lottery *B*: (1000, 0.10; 500 000, 0.89, 0, 0.01).

Choice 2:

Lottery *A*: (5 000 000, 0.11; 0, 0.89);

Lottery *B*: (1 000 000, 0.10; 0, 0.90).

Most individuals prefer the lottery $1A$ to the lottery $1B$ in the first choice and the lottery $2B$ to the lottery $2A$ in the second choice. Such choices are at odds with expected utility theory, which states that the 'right' choices should be either $1A \succ 1B$ and $2A \succ 2B$, or $1B \succ 1A$ and $2B \succ 2A$. (The explanation for these choices is the same as that presented when considering the general ratio effect.)

There have been a large number of experimental studies such as those mentioned above. Virtually all of these studies have found irrefutable proof that, in real-world situations of risky choice, individuals do not follow the prescriptions of expected utility theory. They do not focus on maximising expected utility, but are guided in their choices by estimates of the prospect's outcomes and, most importantly, estimates of the probabilities of these outcomes. These features of real choices in risky situations have led to the development of descriptive choice theory alternative to expected utility theory.

Let us refer to [10] in which the authors give the following example. Let us imagine the following choice situation in which one must choose between being guaranteed $1 000 000 and receiving $5 000 000 with probability of 0.98. Most individuals prefer the first alternative. According to expected utility theory, this means that $u(1\,000\,000) \times 1 > u(5\,000\,000) \times 0.98$. Now let us multiply the values of the relevant probabilities by 0.01. We have the following choice situation: get $1 000 000 with probability of 0.01 or get $5 000 000 with probability of 0.098.

According to the independence of choice from general ratio, which is postulated in expected utility theory, individuals should choose the first alternative in the new choice situation. However, most individuals choose the second alternative. This is a clear violation of the independence of choice from the general ratio and explicitly suggests that the outcomes of choices in risky situations are influenced by the probabilities of the prospect's outcomes.

Other examples of violations of the requirements of expected utility theory in real choice situations are presented in [11]–[14].

In what ways do subjective perceptions of probabilities in risky choice situations occur? Let us refer to the work of [15]. Individuals were asked to choose among simple lotteries $(x,p; 0,1-p)$. Based on the analysis of the election results, it was concluded that individuals exhibited four-way patterns of attitudes towards risk (Table I). In Table I, $c(.,.)$ is the average value of the deterministic equivalent of individuals for each of the prospect types. It is clear from the table that individuals exhibit a tendency to take risks for wins at low probabilities and for losses at high probabilities. On the other hand, individuals are risk averse for wins with high probabilities and risk averse for losses with low probabilities.

TABLE I

FOUR-WAY PATTERNS OF RISK TAKING ATTITUDES (SOURCE [15] WITH REFERENCE TO ANOTHER SOURCE)

	Wins	Losses
Low probability	$C(100, 0.5) = 14$ (tendency to take risks)	$C(-100, 0.05) = -8$ (risk aversion)
High probability	$C(100, 0.95) = 78$ (risk aversion)	$C(-100, 0.05) = -84$ (tendency to take risks)

These results are fully consistent with those of many other empirical studies.

As expected, utility theory appears to be an unsatisfactory descriptive tool for decision-making under risk; the need to develop decision-making theories that can model and account for the actual choices of individuals outside the strict normative requirements of expected utility theory has become clear. First and foremost, it is a matter of correctly accounting for subjective perceptions of the probabilities of prospective outcomes. The idea behind this is to convert (transform) real probability estimates into subjective weights that reflect individuals' personal perceptions of these probabilities when choosing prospects. In other words, the original probability estimates should be weighted. It does not matter whether these original estimates are derived objectively or from subjective expert judgement.

IV. PROBABILITY WEIGHTING FUNCTIONS

A large number of parametric forms of probability weighting functions have been proposed. An overview and analysis of some common forms of these functions can be found in [10]–

[16]. At the very beginning of the development of probability weighting approaches, a power form of the weighting function was proposed

$$w(p) = p^{\gamma}, \ \gamma > 1 . \quad (4)$$

This function is not at all suitable for correct probability weighting. It is sub-proportional over the entire probability range [0, 1] and does not reflect the actual subjective perception of probabilities in risky choices.

Let us consider other common parametric probability weighting functions.

- *Karmakar's weighting function* [17]

$$w(p) = \frac{p^{\gamma}}{p^{\gamma} + (1+p)^{\gamma}}, \ 0 < \gamma < 1 . \quad (5)$$

A characteristic feature of the Karmakar's weighting function is that in the range of probability values [0, 0.5] the weights exceed the values of the corresponding probabilities (over-weighting of small probability values); in the range of probability values [0.5, 1] the weights are smaller than the corresponding probability values (under-weighting of large probability values). This nature of the weighting function, in general, reflects individuals' subjective perceptions of probabilities. It should be noted that the transition from overweighting to underweighting occurs at $p = 0.5$.

- *Prelec-I probability weighting function* [11]

$$w(p) = \frac{1}{e^{(-\ln p)^{\alpha}}}, \ 0 < \alpha < 1 . \quad (6)$$

The parameter α is a measure of the subproportionality of the probability weights. The smaller the value α is, the more the graph of the weighting function deviates from the diagonal line (reduction in subproportionality). When $\alpha = 1$, the weighting function is $w(p) = p$. The graphs of the weighting functions intersect the diagonal line at $p = \frac{1}{e} = 0.368$.

- *Prelec-II probability weighting function* [11]

$$w(p) = \frac{1}{e^{\beta(-\ln p)^{\alpha}}}, \ 0 < \alpha < 1, \ \beta > 0 . \quad (7)$$

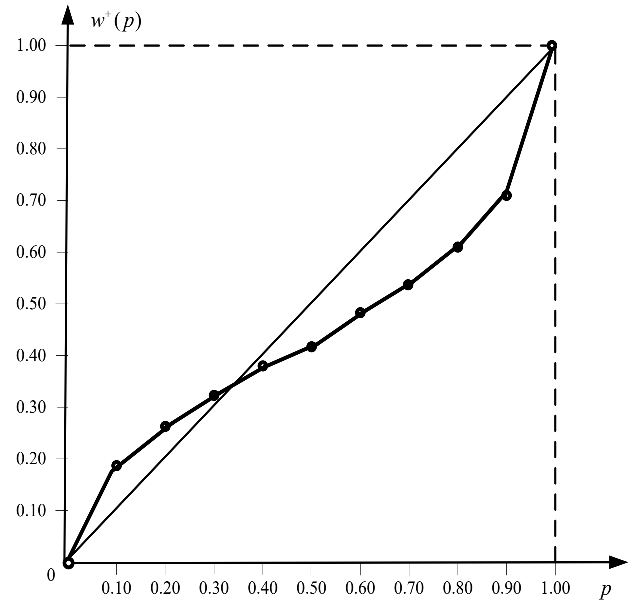
As in the Prelec-I function, the parameter α controls the degree of convexity and concavity of the function $w(p)$. Note that unlike the Prelec-I function, the graphs of the Prelec-II function cross the diagonal line at different points. This makes the weighting function (7) a more flexible tool for representing individuals' subjective perceptions of probabilities.

In [18], the authors suggest using the following probability weighting functions for the probabilities of outcomes with positive and negative estimates.

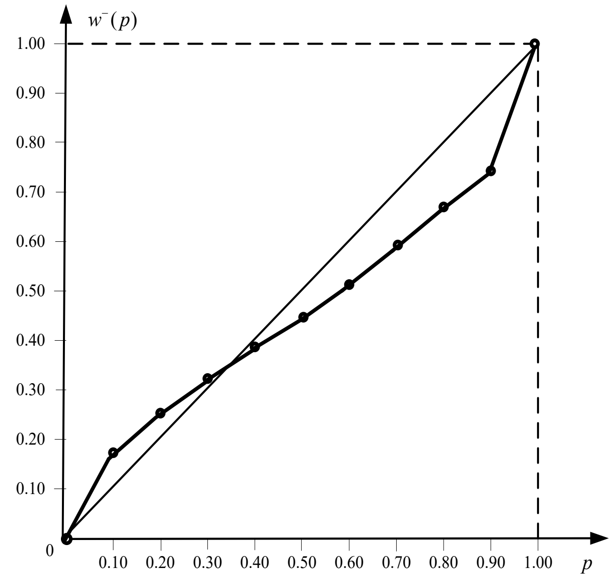
$$w^{+}(p) = \frac{p^{\gamma}}{\left(p^{\gamma} + (1-p)^{\gamma}\right)^{\frac{1}{\gamma}}}, \ \gamma = 0.61 , \quad (8)$$

$$w^{-}(p) = \frac{p^{\sigma}}{\left(p^{\sigma} + (1-p)^{\sigma}\right)^{\frac{1}{\sigma}}}, \ \sigma = 0.69 . \quad (9)$$

Plots of the probability weighting functions (8) and (9) are shown in Fig. 1.



a)



b)

Fig. 1. Plots of the probability weighting functions (8), (9) [18].

The most practical uses are the Prelec-II probability weighting function (7) and the Tversky-Kahneman probability weighting functions (8) and (9).

V. Conclusion

Expected utility theory [1] is based on the use of utility estimates of the outcomes of alternative decisions instead of direct criterion estimates of these outcomes. This approach allows us to model more accurately the subjective preferences of decision makers and their attitudes to risk. For a long time, the expected utility maximisation approach has been the leading approach to choosing actions under risk.

One of the main shortcomings of the expected utility maximisation approach is that the probabilities of relevant events and the corresponding outcomes of alternative decisions were used only as weights in calculating expected utility. In other words, it was implicitly argued that individuals in risky choice situations focused only on maximising their expected utility without considering the probabilities of outcomes.

Practical research into the behaviour of individuals in risky situations has shown with certainty that, in reality, individuals are guided in their choices by the probabilities of the outcomes of alternative decisions. The accumulated results of numerous studies have convincingly demonstrated the need to consider and make practical use of individuals' subjective perceptions of relevant probabilities in their elections. As a result of extensive research, various probability weighting functions have been proposed. Using these functions, the relevant probabilities were transformed into weighting coefficients, which were used in place of the original probability estimates in calculating the expected utility of alternative choices.

Another direction of research in the field of decision-making has been concerned with the subjective evaluation by individuals of the relative importance of the outcomes of alternatives. This led to the creation and development of rank-dependent utility [19].

The generalisation of all these results led to the creation of cumulative prospect theory [18], which combined expected utility theory, the probability weighting principle and rank-dependent utility theory. Nowadays, this theory is the most powerful approach to decision-making under risk.

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