

Adaptive Fuzzy Clustering of Short Time Series with Unevenly Distributed Observations in Data Stream Mining Tasks

Yevgeniy Bodyanskiy¹, Olena Vynokurova², Ilya Kobylin³, Oleg Kobylin⁴
¹⁻⁴Kharkiv National University of Radio Electronics

Abstract – In the paper, adaptive modifications of fuzzy clustering methods have been proposed for solving the problem of data stream mining in online mode. The clustering-segmentation task of short time series with unevenly distributed observations (at the same time in all samples) is considered. The proposed approach for adaptive fuzzy clustering of data stream is sufficiently simple in numerical implementation and is characterised by a high speed of information processing. The computational experiments have confirmed the effectiveness of the developed approach.

Keywords – Data mining, fuzzy clustering methods, hybrid intelligent systems.

I. INTRODUCTION

The tasks of clustering and segmentation of time series are frequently met in data mining [1]–[4], and a lot of methods, including algorithms of online information processing when data are fed in sequential mode, have been proposed for their solution [5]–[7].

However, there are situations in many practical applications, when “classical” approaches to the analysis of time series are not efficient. One of such tasks is fuzzy clustering of short time series with experimental observations that are non-uniformly distributed in time [8].

The problem becomes more complicated by the fact that small data set size does not allow using standard statistic methods because the objects of clusterisation are not individual experimental observations but samples as a whole. Observations are made in unevenly distributed instants of time (at the same time in all samples), and the formed clusters overlap so that each series realisation may belong to several classes at a time.

It is also assumed that all the processed information is given in a batch form, and its volume cannot change over time.

It seems reasonable to spread the approach introduced in [8] in a situation where data are fed to processing in online mode in the form of the flow of information within the concept of data stream mining [9], [10].

II. ADAPTIVE PROBABILISTIC FUZZY CLUSTERING OF SHORT SEQUENCES

That’s supposed that the initial information is given in the form of a set of samples $x_i(k)$ (here $i = 1, 2, \dots, n$ is a number of an individual observation in k -th realisation, $k = 1, 2, \dots, N$),

which contains $N(N > n)$ time series with an uneven quantisation of time-step subject to clustering; wherein each such instance may be represented in the form of $(n \times 1)$ vector $x(k) = (x_1(k), x_2(k), \dots, x_n(k))^T$. Non-uniformity of quantisation means that

$$\Delta t_i = t_i - t_{i-1} \neq \Delta t_{i+1} = t_{i+1} - t_i,$$

i.e., $\Delta t_i \neq \text{const}$.

Figure 1 shows an example of such realisation. It is obvious that neither traditional Euclidean metric nor classical stochastic criteria can be used to estimate the distance between such samples. In this connection, similarity measure of time series PS-distance (Piecewise slope distance = PS – distance = STS – distance = short time series distance) has been introduced in [8], based on representing of these series as piecewise linear functions

$$x_i(k) = a_i(k) + b_i(k)t \quad (1)$$

where $t_i \leq t \leq t_{i+1}$,

$$\begin{cases} a_i(k) = \frac{t_{i+1}x_i(k) - t_i x_{i+1}(k)}{t_{i+1} - t_i}, \\ b_i(k) = \frac{x_{i+1}(k) - x_i(k)}{t_{i+1} - t_i} \end{cases}$$

where $x_i(k)$ is truncated notation of $x_i(k)$ (value of signal in t_i moment) and estimating difference of forms (slopes) of analysed samples.

The distance between two sequences $x(k)$ and $x(l)$ can be written in the form of expression:

$$\begin{aligned} d_{\text{STS}}^2(x(k), x(l)) &= \\ &= \sum_{i=1}^{n-1} \left(\frac{x_{i+1}(k) - x_i(k)}{t_{i+1} - t_i} - \frac{x_{i+1}(l) - x_i(l)}{t_{i+1} - t_i} \right)^2 = \\ &= \sum_{i=1}^{n-1} \left(\frac{x_{i+1}(k) - x_i(k)}{\Delta t_{i+1}} - \frac{x_{i+1}(l) - x_i(l)}{\Delta t_{i+1}} \right)^2, \end{aligned} \quad (2)$$

satisfying all the conditions that determine the metric.

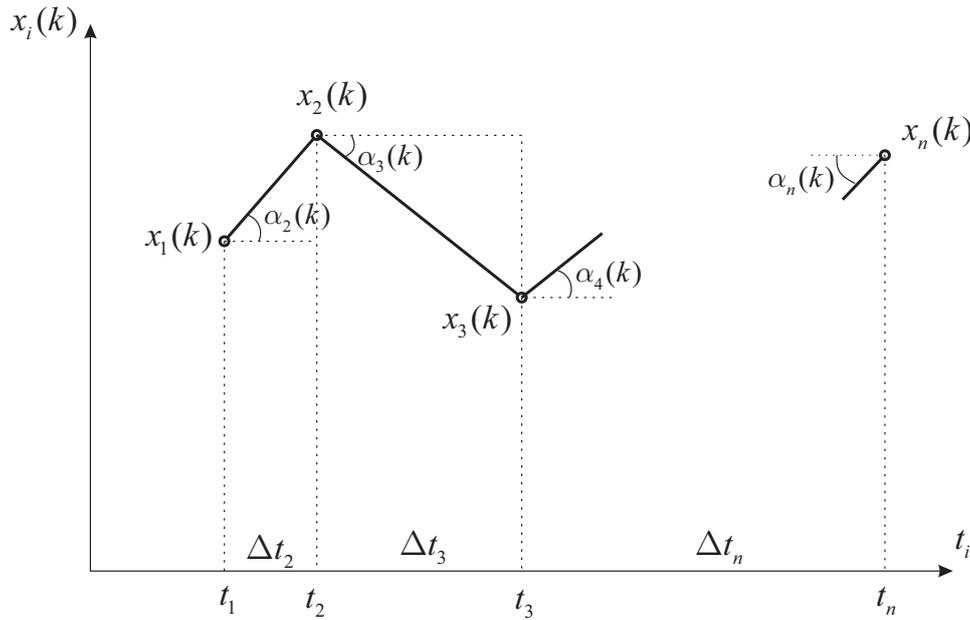


Fig. 1. Time series with non-uniform quantisation tact.

On the basis of metric (2), the authors [8] have introduced a batch (off-line) fuzzy clustering procedure, which is the modification (somewhat tedious) of fuzzy c-means (FCM) algorithm in the situation of processing time series with unevenly distributed observations.

It is easy to notice that the components of expression (2) are nothing more than the first differences of digital signal $x_i(k)$ or tangents of linear function (1) angles, i.e.:

$$\Delta x_{i+1}(k) = \frac{x_{i+1}(k) - x_i(k)}{\Delta t_{i+1}} = \operatorname{tg} \alpha_{i+1}(k).$$

However, a sequence formed by the first differences contains one point less than the initial sample, i.e., $(n-1)$ observations $\Delta x_2(k), \Delta x_3(k), \dots, \Delta x_n(k)$, or $\operatorname{tg} \alpha_2(k), \operatorname{tg} \alpha_3(k), \dots, \operatorname{tg} \alpha_n(k)$. As a result of taking series differences from the average value, its mean value is removed. To restore the original sampling rate by its differences, it is necessary to add a set of these differences with any of the observations of initial sequence, for example, $x_n(k)$. Then, having the sequence of differences $\Delta x_i(k)$, it is not difficult to restore the original sequence using simple expressions:

$$\begin{cases} x_{n-1}(k) = x_n(k) - \Delta x_n(k) \Delta t_n, \\ x_{n-2}(k) = x_n(k) - \Delta x_{n-1}(k) \Delta t_{n-1}, \\ \vdots \\ x_1(k) = x_2(k) - \Delta x_2(k) \Delta t_2. \end{cases} \quad (3)$$

Then by taking into consideration $(n \times 1)$ -vector of features $\tilde{x}(k) = (\Delta x_2(k), \Delta x_3(k), \dots, \Delta x_n(k), x_n(k))^T$ it is easy to rewrite metric (2) in a traditional form:

$$d_{\text{STS}}^2(x(k), x(l)) = \|\tilde{x}(k) - \tilde{x}(l)\|^2, \quad (4)$$

i.e., to return, in fact, to standard Euclidean distance between the differences of the original series. Further, using metric (4) and the standard technique of fuzzy probabilistic cluster analysis, it is possible to find the saddle point of Lagrangian function:

$$\begin{aligned} L(u_j(k), \tilde{c}_j, \lambda(k)) &= \\ &= \sum_{k=1}^N \sum_{j=1}^m u_j^\beta(k) \|\tilde{x}(k) - \tilde{c}_j\|^2 + \\ &+ \sum_{k=1}^N \lambda(k) \left(\sum_{j=1}^m u_j(k) - 1 \right) \end{aligned} \quad (5)$$

where $u_j(k)$ – the membership level of vector $\tilde{x}(k)$ to j cluster with centroid – prototype \tilde{c}_j , $j = 1, 2, \dots, m$; m is a number of clusters defined a priori; $\lambda(k)$ is the undetermined Lagrange multiplier, $\beta > 1$ fuzzifier that defines the “blurring” of boundaries between clusters. Thus, we come to the standard procedure of fuzzy probabilistic clustering:

$$\begin{cases} u_j(k) = \frac{\left(\|\tilde{x}(k) - \tilde{c}_j\|^2 \right)^{\frac{1}{1-\beta}}}{\sum_{l=1}^m \left(\|\tilde{x}(k) - \tilde{c}_l\|^2 \right)^{\frac{1}{1-\beta}}}, \\ \tilde{c}_j = \frac{\sum_{k=1}^N u_j^\beta(k) \tilde{x}(k)}{\sum_{k=1}^N u_j^\beta(k)} \end{cases} \quad (6)$$

which at $\beta = 2$ coincides with the popular J. Bezdek's FCM algorithm:

$$\begin{cases} u_j(k) = \frac{\|\tilde{x}(k) - \tilde{c}_j\|^{-2}}{\sum_{l=1}^m \|\tilde{x}(k) - \tilde{c}_l\|^{-2}}, \\ \tilde{c}_j = \frac{\sum_{k=1}^N u_j^2(k) \tilde{x}(k)}{\sum_{k=1}^N u_j^2(k)}. \end{cases} \quad (7)$$

Since vector $\tilde{c}_j, j = 1, 2, \dots, m$ of cluster are centroids formed by series of differences in order to restore the original data prototypes c_j relations (3) can be used.

Clustering procedures (5) and (6) have been synthesised on the assumption that all of the original information is set as a fixed data array $x(1), x(2), \dots, x(N)$ and it does not change during processing. If samples $x(k)$ are sequentially fed to processing in the form of a data stream, we can use approaches used in data stream mining and dynamic data mining and, first of all, adaptive methods.

Using Arrow-Hurwicz-Uzawa recursive algorithm of nonlinear programming for searching of Lagrangian (5) saddle point, we obtain the adaptive gradient procedures of fuzzy clustering [11]:

$$\begin{cases} u_j(k+1) = \frac{\left(\|\tilde{x}(k+1) - \tilde{c}_j(k)\|^2\right)^{\frac{1}{1-\beta}}}{\sum_{l=1}^m \left(\|\tilde{x}(k) - \tilde{c}_l\|^2\right)^{\frac{1}{1-\beta}}}, \\ \tilde{c}_j(k+1) = \tilde{c}_j(k) + \eta(k) u_j^\beta(k+1) (\tilde{x}(k+1) - \tilde{c}_j(k)) \end{cases} \quad (8)$$

where $\eta(k)$ is a learning step parameter.

It is understood that during information processing each newly arriving data vector $x(k+1)$ by taking the difference is converted into $\tilde{x}(k+1)$, and the obtained centroids $\tilde{c}_j(k+1)$ should be translated into prototypes $\tilde{c}_j(k+1)$.

It is interesting to note that from the point of view of self-organising Kohonen maps learning [12], the second recurrent relation (8) is a self-learning rule based on the principle "Winner Takes More" (WTM), and factor $u_j^\beta(k+1)$ corresponds to a neighbouring function of Cauchy form instead of the traditional Gaussian [13]. It is also clear that at $\beta = 0$ we come to the standard WTA ("Winner Takes All") self-learning rule:

$$\tilde{c}_j(k+1) = \tilde{c}_j(k) + \eta(k) (\tilde{x}(k+1) - \tilde{c}_j(k)),$$

which minimises the objective function:

$$E(\tilde{c}_j) = \sum_k \|\tilde{x}(k) - c_j\|^2.$$

When $\eta(k) = (k+1)^{-1}$ we come to a stochastic approximation procedure:

$$\tilde{c}_j(k+1) = \tilde{c}_j(k) + \frac{1}{k+1} (\tilde{x}(k+1) - \tilde{c}_j(k)),$$

leading to a standard estimation of the arithmetic mean as the centroid.

Thus, to solve the problem of fuzzy clustering of short time series with an uneven fact quantisation in online mode, a numerically simple adaptive algorithm (8) can be used, which is the extension of WTM-rule of Kohonen's self-learning on the problem under consideration.

III. ADAPTIVE POSSIBILISTIC FUZZY CLUSTERING OF SHORT SAMPLES

Despite their widespread occurrence, algorithms associated with the optimisation of Lagrangian (5) also have a major drawback as they should perform the constrain:

$$\sum_{j=1}^m u_j(k) = 1. \quad (9)$$

Specifically, due to constraint (9), these procedures are called probabilistic. The drawback itself arising from (9) consists in the fact that observation vector $\tilde{x}(k)$, equally belonging to all clusters, has the same levels of membership as the vector that does not belong to any class, but is equidistant from all the centroids. Thus, the abnormal outlier will belong to all available clusters.

An alternative of probabilistic clustering algorithms is possibilistic methods [14], which are related to the minimisation of the objective function:

$$\begin{aligned} E(u_j(k), \tilde{c}_j, \mu_j) &= \sum_{j=1}^m \mu_j \sum_{k=1}^N (1 - u_j(k))^\beta + \\ &+ \sum_{k=1}^N \sum_{j=1}^m u_j^\beta(k) \|\tilde{x}(k) - \tilde{c}_j\|^2 \end{aligned}, \quad (10)$$

where $\mu_j > 0$ determines the distance from $\tilde{x}(k)$ to \tilde{c}_j at which the membership level takes a value of 0.5, i.e.

$$u_j(k) = 0,5, \text{ when } \|\tilde{x}(k) - \tilde{c}_j\|^2 = \mu_j.$$

Optimisation (10) on $u_j(k)$, \tilde{c}_j and μ_j leads to a result in the form:

$$\left\{ \begin{array}{l} u_j(k) = \left(1 + \left(\frac{\|\tilde{x}(k) - \tilde{c}_j\|^2}{\mu_j} \right)^{\frac{1}{\beta-1}} \right)^{-1}, \\ \mu_j = \frac{\sum_{k=1}^N u_j^\beta(k) \|\tilde{x}(k) - \tilde{c}_j\|^2}{\sum_{k=1}^N u_j^\beta(k)}, \\ \tilde{c}_j = \frac{\sum_{k=1}^N u_j^\beta(k) \tilde{x}(k)}{\sum_{k=1}^N u_j^\beta(k)}. \end{array} \right. \quad (11)$$

It is interesting to note in this regard that the expressions for calculation of centroids in (6) and (11) coincide.

Adaptive version of possibilistic algorithm (11) can be obtained by gradient optimisation of the objective function (10) in the form of [13]:

$$\left\{ \begin{array}{l} u_j(k+1) = \left(1 + \left(\frac{\|\tilde{x}(k+1) - \tilde{c}_j(k)\|^2}{\mu_j(k)} \right)^{\frac{1}{\beta-1}} \right)^{-1}, \\ \mu_j(k+1) = \frac{\sum_{p=1}^{k+1} u_j^\beta(p) \|\tilde{x}(p) - \tilde{c}_j(k)\|^2}{\sum_{p=1}^{k+1} u_j^\beta(p)}, \\ \tilde{c}_j(k+1) = \tilde{c}_j(k) + \eta(k) u_j^\beta(k+1) (\tilde{x}(k+1) - \tilde{c}_j(k)), \end{array} \right. \quad (12)$$

where the third relation is also the WTM self-learning, but differs from a similar expressive in (7) by the type of neighbourhood function.

If the information processing detects that a certain observation vector has small levels of membership to any of the clusters, this indicates that these observations are either irregular outlier or it indicates the occurrence of a new cluster, which is different from the already existing ones.

IV. EXPERIMENTAL RESULTS

For the efficiency confirmation of the proposed approach to clustering-segmentation of short time series with unevenly distributed observations, the task of clustering-segmentation of hourly energy consumption time series was considered. Results of the proposed approach allow increasing the quality of analysis and prediction of time series.

Time series consists of 2400 observations. For clustering this time series was divided by segments with 8 observations. For obtaining unevenly distributed observations in each segment, the 3rd and 5th observations were deleted from a data set.

Hence, according to (3) we obtain the data set in form of the table "object-properties" with 300 observations and 6 properties. A number of clusters were $m = 3$ (morning, day and evening segments of energy consumption).

All clustering algorithms were tested by the same data set. Average mean class error (MCE) was taken as the quality criterion of clustering results.

In the first experiment, we compared the performance of the clustering algorithms in the problem of classification when instances of all the available classes were present in the data set used for clustering, i.e., the number of classes was known a priori and equal to 3. The data sets were divided into the training and testing sets with 70 % and 30 % of data, respectively. For better performance of the recursive clustering algorithms, the data sets were randomly shuffled. The training sets were used for the initialisation of the classifier through fuzzy clustering, and the testing sets were used for the comparison of the classification accuracy. We used the learning rate $\eta = 0.01$ in the recursive procedures (8) and (12), and the "fuzzifier" parameter was taken $\beta = 1.1$. Both possibilistic procedures (batch and recursive) were initialised from the results of probabilistic clustering through the fuzzy c-means algorithm. We performed 10 iterations for the batch clustering procedures, and 10 runs over the training data for the recursive clustering procedures. The experiment was repeated 50 times, and then average results were calculated. The results are given in Table I. They represent the percentage of the incorrectly classified objects from the testing data set.

TABLE I
RESULTS OF CLUSTERING-SEGMENTATION OF TIME SERIES

CLUSTERING PROCEDURES	M{MCE}
Fuzzy probabilistic clustering algorithm	1.6 % (5)
Adaptive fuzzy probabilistic clustering algorithm	1.3 % (4)
Possibilistic clustering algorithm	11.1 % (33)
Adaptive possibilistic clustering algorithm	6.3 % (19)

Figures 2 and 3 show the clustering results, which were obtained by the proposed approaches.

As it can be seen from the obtained results, the fuzzy probabilistic clustering algorithms have the best quality of clustering (both batch and adaptive modes). We can also see that the results of adaptive modes of clustering algorithms have better quality than that of batch mode.

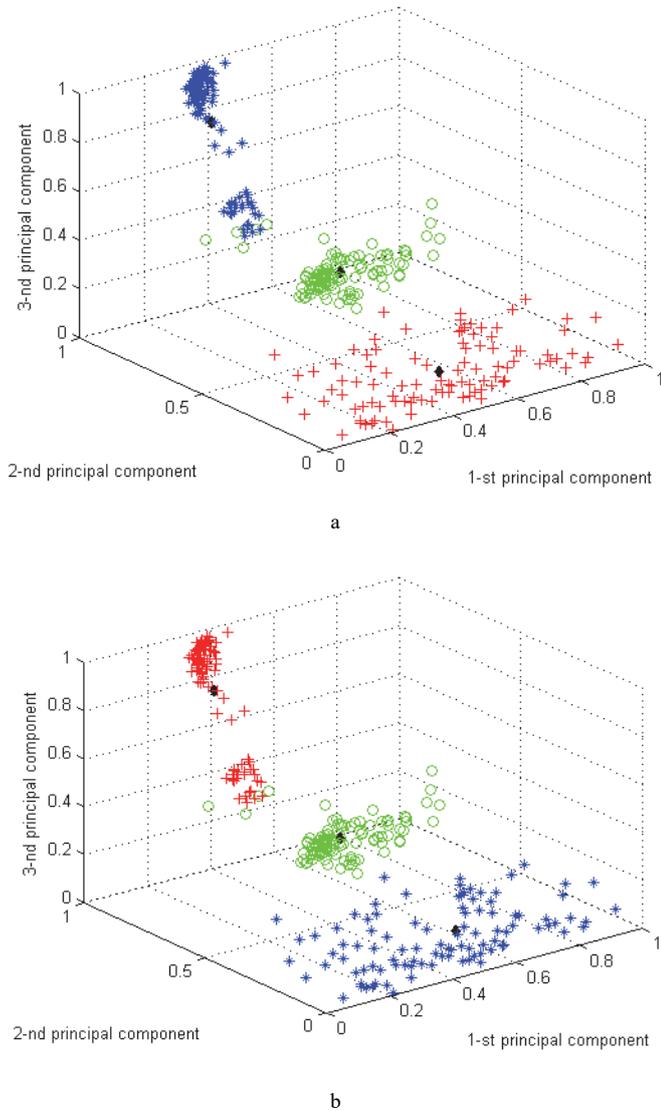


Fig. 2. The projection of data set and prototypes of clusters on principal components for fuzzy probabilistic clustering algorithms (a – batch mode, b – adaptive mode).

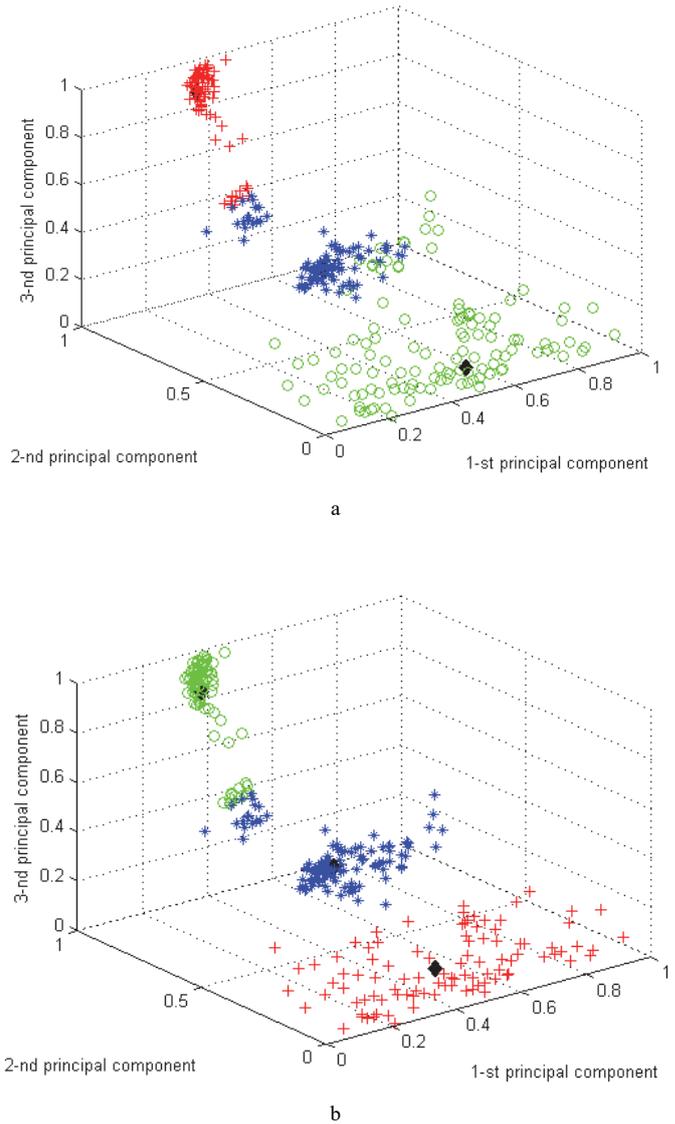


Fig. 3. The projection of data set and prototypes of clusters on principal components for possibilistic clustering algorithms (a – batch mode, b – adaptive mode).

V. CONCLUSION

The authors of the present research have investigated the problem of fuzzy clustering of short time series with unevenly distributed observations, where the information analysis is implemented sequentially in online mode. Adaptive modifications of probabilistic and possibilistic clustering methods have been introduced focusing on solving the problem. Being essentially a kind of T. Kohonen’s WTM self-learning rules, the proposed procedures provide simple numerical implementation and high-speed information processing. The methods under consideration have shown better performance than the conventional algorithms in clustering unevenly sampled short time-series data. The computational experiments based on both benchmarks and real data sets have confirmed the effectiveness of the developed approach.

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Yevgeniy Bodyanskiy. In 1971, he graduated with honour from Kharkiv National University of Radio Electronics. In 1980, he defended the Doctoral Thesis. In 1984, he was awarded the academic title of Senior Researcher. In 1990, he was awarded Dr. habil. sc. ing. degree. In 1994, he was awarded the academic title of Professor. His major fields of research are evolving hybrid systems of computational intelligence, data stream mining, data science, and big data.

Since 1974 he has been working at Kharkiv National University of Radio Electronics. In 1974–1976, he was a Researcher; in 1977–1983, he was a Senior Researcher; in 1986–1991, he was a Scientific Head of Control Systems Research Laboratory; in 1991–1992, he was a Research Fellow. Since 1992 he has been a Professor of Artificial Intelligence Department at KhNURE,

Scientific Head of Control Systems Research Laboratory at KhNURE. He has more than 660 scientific publications, including 42 inventions and 16 monographs. Research interests include hybrid systems of computational intelligence: adaptive, neuro-, wavelet-, neo-fuzzy-, real-time systems, including problems connected with control, identification, forecasting, clustering, diagnostics, fault detection in technical, economic, medical and ecological objects.

He is the IEEE senior member, member of 4 scientific and 7 editorial boards.

Address: Office 511, Nauky Ave., 14, Kharkiv, 61166, Ukraine.

E-mail: yevgeniy.bodyanskiy@nure.ua

Olena Vynokurova. In 2002, she graduated with honour from Kharkiv National University of Radio Electronics. From 2002 to 2005, she undertook the postgraduate study at the Department of Artificial Intelligence. In 2005, she defended the Doctoral Thesis. In 2007, she was awarded the academic title of Senior Researcher. In 2012, she was awarded Dr. habil. sc. ing. In 2014, she was awarded the academic title of Professor. Her major fields of research are evolving hybrid neuro-fuzzy systems for dynamic data mining, data stream mining.

Since 2002 she has been working at Kharkiv National University of Radio Electronics. In 2002–2005, she was a Researcher; in 2005–2010, she was a Senior Researcher; in 2010–2014, she was a Leading Researcher. Since 2014 she has been a Leading Researcher of Control Systems Research Laboratory; since 2013 she has been a Professor of Information Technology Security Department at Kharkiv National University of Radio Electronics. She has more than 150 scientific publications, including 6 monographs. Research interests include evolving hybrid systems of computational intelligence: wavelet neural networks, hybrid wavelet neuro-fuzzy systems, identification, forecasting, clustering, diagnostics, fault detection in technical, economic, medical and ecological objects.

Address: Office 517, Nauky Ave., 14, Kharkiv, 61166, Ukraine.

E-mail: vynokurova@gmail.com

Ilya Kobylin. In 2015, he graduated with honour from Kharkiv National University of Radio Electronics. He is a Doctoral student at Kharkiv National University of Radio Electronics. His major fields of research are hybrid neuro-fuzzy systems for data mining problems.

He has 5 scientific publications. Research interests include hybrid neuro-fuzzy systems, clustering.

Address: Office 517, Nauky Ave., 14, Kharkiv, 61166, Ukraine.

E-mail: ilya.kobylin@nure.ua

Oleg Kobylin. In 1995, he graduated from Kharkiv State Technical University of Radio Electronics, the School of Computer Engineering.

In 2007, he completed the postgraduate studies. In 2007 he defended the Doctoral Thesis.

Since 2000 he has been working at Kharkiv National University of Radio Electronics.

Since 2010 he has been an Associate Professor at the Department of Informatics of Kharkiv National University of Radio Electronics.

He has more than 40 scientific publications. Research interests include evolving hybrid systems of computational intelligence:

image segmentation, spectral image analysis, identification, forecasting, clustering, and diagnostics.

Address: Office 288, Nauky Ave., 14, Kharkiv, 61166, Ukraine.

E-mail: oleg.kobylin@gmail.com