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Evaluation of Payoff Matrices for Non-Cooperative Games via Processing Binary Expert Estimations

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Abstract **– A problem of evaluating the non-cooperative game model is considered in the paper. The evaluation is understood in the sense of obtaining the game payoff matrices whose entries are single-point values. Experts participating in the estimation procedure make their judgments on all the game situations for every player. A form of expert estimations is suggested. The form is of binary type, wherein the expert's judgment is either 1 or 0. This type is the easiest to be implemented in social networks. For most social networks, 1 can be a "like" (the currently evaluated situation is advantageous), and 0 is a "dislike" (disadvantageous). A method of processing expert estimations is substantiated. Two requirements are provided for obtaining disambiguous payoff averages along with the clustered payoff matrices.**

Keywords **– Estimation procedure, expert's binary judgment, non-cooperative game, payoff averages, payoff matrice evaluation.**

I. Introduction

An ambiguous problem in the game theory and decision making is numerical evaluation of situations [1], [2]. A situation, in those fields, is a list of pure strategies (decisions and states [3], [4]). Commonly, there are no theoretic ways to evaluate all the set of pure strategy situations and to get payoff matrices outright [2], [5]. The only theorised way is to use binary relations [6], [7]. This leads to the simplest pairwise comparisons, where experts are nonetheless required [8], [9]. The question is how many experts should be invoked and what method should be applied to process the expert estimations.

II. Analysis of Related Research

The number of experts depends on the field of study. Sometimes the field is so difficult that a very limited number of experts can be recruited [10], [11]. Competence of experts influences their number as well. The less proficient experts are, the greater number of them is required. Recently, social networks suggested a powerful means to collect data, including opinions, judgments, estimations which are put by users [12].

Methods of processing expert estimations are determined by their form and structure. Common statistical methods fit for single-point estimations. If the expert estimation is a bunch of single-point estimations that are supposed to have interconnections, then simple averaging is not acceptable [13], [14]. Instead of finding usual math expectance and minimising variance, a consensus estimation is searched by setting it as close to every expert estimation as possible [15], [16]. Closeness is understood in the sense of one of the following metrics: Euclidean, Manhattan, Cosine, Dice, and Jaccard [17], [18]. However, in the non-cooperative game theory, payoff evaluation procedures still do not have a strict algorithm.

III. Goal and Items to be Accomplished

Thus, an approach of evaluating payoff matrices in the non-cooperative game is needed. The goal is to formalise the whole evaluation procedure – since experts' judgments come with the ready players' payoff matrices. This goal is intended to be reached after accomplishing the three items:

1. Suggestion of a form of expert estimations.
2. Substantiation of how many experts are needed.
3. Substantiation of a method of processing expert estimations.

Obviously, item #3 influences item #2 and vice versa. Items #1 and #2 are also interrelated, but the number of experts is crucial, whatever the estimation form is.

IV. The Form of Expert Estimations

The form of expert estimations should be the simplest if they are put through social networks. This is explained with that, even for dyadic games [19] of three persons, an expert is obliged to estimate 24 situations (8 situations for every player in such dyadic games). Therefore, each situation must be estimated quickly to process all of them in time and not to get confused. The simplest form of expert estimations is binary, where the expert's judgment is equivalent to the value 0 or 1. For most social networks, 1 can be a "like" (the currently evaluated situation is advantageous), and 0 is a "dislike" (disadvantageous).

V. Number of Experts Needed for Estimation Procedure

The number of experts is determined by their qualification and proficiency (confidence), and also depends on the study complication [10], [11], [15], [20]. As the study complication grows, the number of experts should be increased [11], [21], [22]. A factor of complication is the number of situations which are going to be estimated (liked/disliked).

To obtain adequate estimation results, at least ten (several tens of) experts are always needed. This is for 2×2 or $2 \times 2 \times 2$ games only. A hundred (or several hundred) experts is needed for games of bigger formats, where either the number of players is greater than three, or the number of pure strategies is greater than just two, or both [1], [2], [5], [23], [24].

These requirements can be met to a full extent when experts are users of social (corporative) computer networks. For a pretty short while, the number of users which can be involved into binary estimation is always sufficient. Even without their strong proficiency or confidence, the average of estimations is believed to be close to essence owing to the law of large numbers [25], [26].

VI. Method of Processing Expert Estimations

Denote the i -th player's payoff matrix by $\mathbf{R}_i$. For game of N players, $N \in \mathbb{N} \setminus \{1\}$, we have to know N such matrices. These ones are N -dimensional matrices. Thus, the non-cooperative game is

$$\left\langle \{X_i\}_{i=1}^{N}, \{\mathbf{R}_i\}_{i=1}^{N} \right\rangle \tag{1}$$

by the i -th player's pure strategy set X_i . If cardinality $|X_i| = m_i$, $m_i \in \mathbb{N} \setminus \{1\}$, then tuple (1) is the non-cooperative $\bigtimes_{i=1}^{N} m_i$ -game.

The i -th player's payoff matrix is

$$\mathbf{R}_i = \left(r_J^{\langle i \rangle} \right)_{\mathscr{F}}$$

by its format

$$\mathscr{F} = \bigtimes_{i=1}^{N} m_i$$

and indices' set J . This set is of N elements:

$$J = \{j_k\}_{k=1}^{N} \text{ by } j_k \in \left\{\overline{1, m_k}\right\}.$$

The number of all entries in $\mathbf{R}_i$ is

$$E_N = \prod_{k=1}^{N} m_k$$

that is equal to the total number of pure strategy situations in game (1). If, say, the indices' set J corresponds to the situation

$$\{x_i\}_{i=1}^{N} \text{ by } x_i \in X_i \tag{2}$$

then every expert is going to estimate situation (2) for each of N players. Thus, the expert estimates, during a single procedure, $N \cdot E_N$ situations.

It has been stated previously that the number of experts S should be great enough for binary judgments, especially when experts are not proficient enough. Let the s -th expert estimate situation (2) for the k -th player with its value $b_s(J, k) = 1$ if, for this player, situation (2) is favourable. If it is unfavourable, the s -th expert puts $b_s(J, k) = 0$. His or her own payoff matrix for the k -th player is

$$\mathbf{B}_s^{\langle k \rangle} = \left[b_s(J, k) \right]_{\mathscr{F}}.$$

Evaluation of situation (2) is the average

$$\tilde{r}_J^{\langle k \rangle} = \frac{1}{S} \sum_{s=1}^{S} b_s(J, k), \tag{3}$$

being the respective entry of the matrix

$$\tilde{\mathbf{R}}_k = \left(\tilde{r}_J^{\langle k \rangle} \right)_{\mathscr{F}} = \frac{1}{S} \sum_{s=1}^{S} \mathbf{B}_s^{\langle k \rangle}. \tag{4}$$

At first glance, matrix (4) is an evaluation of the k -th player's payoff matrix $\mathbf{R}_k$. But is it correct to use (4) without any restrictions? Indeed, if (3) is close to 0.5 then it means that about a half of all experts judges situation (2) favourable for the k -th player, whereas the second half judges it unfavourable. Thus, estimation of situation (2) comes uncertain. Closeness to 0.5 may be treated differently depending on integers S , N , and E_N . Naively, value $\tilde{r}_J^{\langle k \rangle} = 0.45$ or $\tilde{r}_J^{\langle k \rangle} = 0.55$ gives much the same uncertainty as $\tilde{r}_J^{\langle k \rangle} = 0.5$ gives. Therefore, value (3) is not going to be counted uncertain (then we will call it certain) if

$$\tilde{r}_J^{\langle k \rangle} \notin [0.5 - \delta; 0.5 + \delta] \text{ by } \delta \in (0; 0.25]. \tag{5}$$

Otherwise, if (5) fails, value (3) is counted uncertain.

Nevertheless, obtaining certain evaluations of all situations is not sufficient for accepting them finally and taking

$$\mathbf{R}_k = \tilde{\mathbf{R}}_k \quad \forall k = \overline{1, N}. \tag{6}$$

Without losing generality, consider a counterexample of bimatrix 2×2 game. Say,

$$\mathbf{B}_{s_1}^{\langle 1 \rangle} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \text{ by } s_1 = \overline{1, 10}$$
$$\text{and } \mathbf{B}_{s_2}^{\langle 1 \rangle} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \text{ by } s_2 = \overline{11, 30}. \tag{7}$$

Here the evaluation of payoff matrix of the first player is

$$\tilde{\mathbf{R}}_1 = \begin{pmatrix} \tilde{r}_{11}^{\langle 1 \rangle} & \tilde{r}_{12}^{\langle 1 \rangle} \\ \tilde{r}_{21}^{\langle 1 \rangle} & \tilde{r}_{22}^{\langle 1 \rangle} \end{pmatrix} = \begin{pmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{pmatrix}$$

whose entries satisfy the condition (5). However, it should be noticed that matrices $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ in (7) are fully contrary – one of them is the inversion of the other, i.e., $\mathbf{B}_{s_1}^{\langle 1 \rangle} = 1 - \mathbf{B}_{s_2}^{\langle 1 \rangle}$. Secondly, the ratio of numbers of the “inverted” matrices is 1:2 – every third expert judges fully contrarily. That is why we should restrict scattering of matrices $\left\{\mathbf{B}_s^{\langle k\rangle}\right\}_{s=1}^{S}$ $\forall k=\overline{1, N}$.

For doing that, firstly, a metric (distance function) in the space of binary hyperparallelepipedic matrices is stated. This space is

$$\mathscr{B}=\left\{\mathbf{B}=\left(b_J\right)_{\mathscr{F}} \mid b_J \in\{0, 1\}\right\} . \tag{8}$$

If

$$\mathbf{X}=\left(x_J\right)_{\mathscr{F}} \in \mathscr{B} \text{ and } \mathbf{Y}=\left(y_J\right)_{\mathscr{F}} \in \mathscr{B}$$

then

$$\rho_{\mathscr{B}}(\mathbf{X}, \mathbf{Y})=\sqrt{\sum_{\substack{j_k=\overline{1, m_k} \\ k=\overline{1, N}}}\left(x_J-y_J\right)^2}=\sqrt{\sum_{\substack{j_k=\overline{1, m_k} \\ k=\overline{1, N}}} \operatorname{sign}\left|x_J-y_J\right|} \tag{9}$$

is the distance between $\mathbf{X}$ and $\mathbf{Y}$. As it is easy to see, distance (9) is maximal when matrices are inversions of each other, i.e., $x_J=1-y_J$ for all J :

$$\max_{\mathbf{X} \in \mathscr{B}, \mathbf{Y} \in \mathscr{B}} \rho_{\mathscr{B}}(\mathbf{X}, \mathbf{Y})=$$
$$=\sqrt{\sum_{\substack{j_k=\overline{1, m_k} \\ k=\overline{1, N}}} \operatorname{sign}|1-0|}=\sqrt{\sum_{\substack{j_k=\overline{1, m_k} \\ k=\overline{1, N}}} 1}=E_N . \tag{10}$$

Optionally, maximal distance between two evaluations may be restricted as follows:

$$\max_{\substack{s_1=\overline{1, S} \\ s_2=\overline{1, S}}} \rho_{\mathscr{B}}\left(\mathbf{B}_{s_1}^{\langle k\rangle}, \mathbf{B}_{s_2}^{\langle k\rangle}\right)<E_N . \tag{11}$$

However, restriction (11) is not principal, especially for dyadic games – remember the example with evaluations (7). The crucial restriction is that evaluations are scattered not much. This is

$$\frac{1}{S} \sum_{s=1}^S \rho_{\tilde{\mathscr{B}}}\left(\mathbf{B}_s^{\langle k\rangle}, \tilde{\mathbf{R}}_k\right) \leqslant \lambda \tag{12}$$

by the positive constant λ which is connected to (10), and the space

$$\tilde{\mathscr{B}}=\left\{\tilde{\mathbf{B}}=\left(\tilde{b}_J\right)_{\mathscr{F}} \mid \tilde{b}_J \in[0 ; 1]\right\} \tag{13}$$

containing any evaluations, wherein

$$\rho_{\tilde{\mathscr{B}}}\left(\tilde{\mathbf{B}}_1, \tilde{\mathbf{B}}_2\right)=\sqrt{\sum_{\substack{j_k=\overline{1, m_k} \\ k=\overline{1, N}}}\left(\tilde{b}_J^{\langle 1\rangle}-\tilde{b}_J^{\langle 2\rangle}\right)^2}$$
$$\text{by } \tilde{\mathbf{B}}_1=\left(\tilde{b}_J^{\langle 1\rangle}\right)_{\mathscr{F}} \in \tilde{\mathscr{B}} \text{ and } \tilde{\mathbf{B}}_2=\left(\tilde{b}_J^{\langle 2\rangle}\right)_{\mathscr{F}} \in \tilde{\mathscr{B}} \tag{14}$$

is the corresponding metric.

The method of processing expert estimations is demonstrated in Fig. 1. Its essentiality lies in giving an evaluation of the non-cooperative game (1), starting with only a number of players N and a number of pure strategies for each of them — these are the integers $\left\{m_i\right\}_{i=1}^N$. These numbers are presumed to be known before. Note that the scheme in Fig. 1 does not specify the form of expert estimations. Besides, evaluation (4) and inequality (12) may be improved, if necessary.

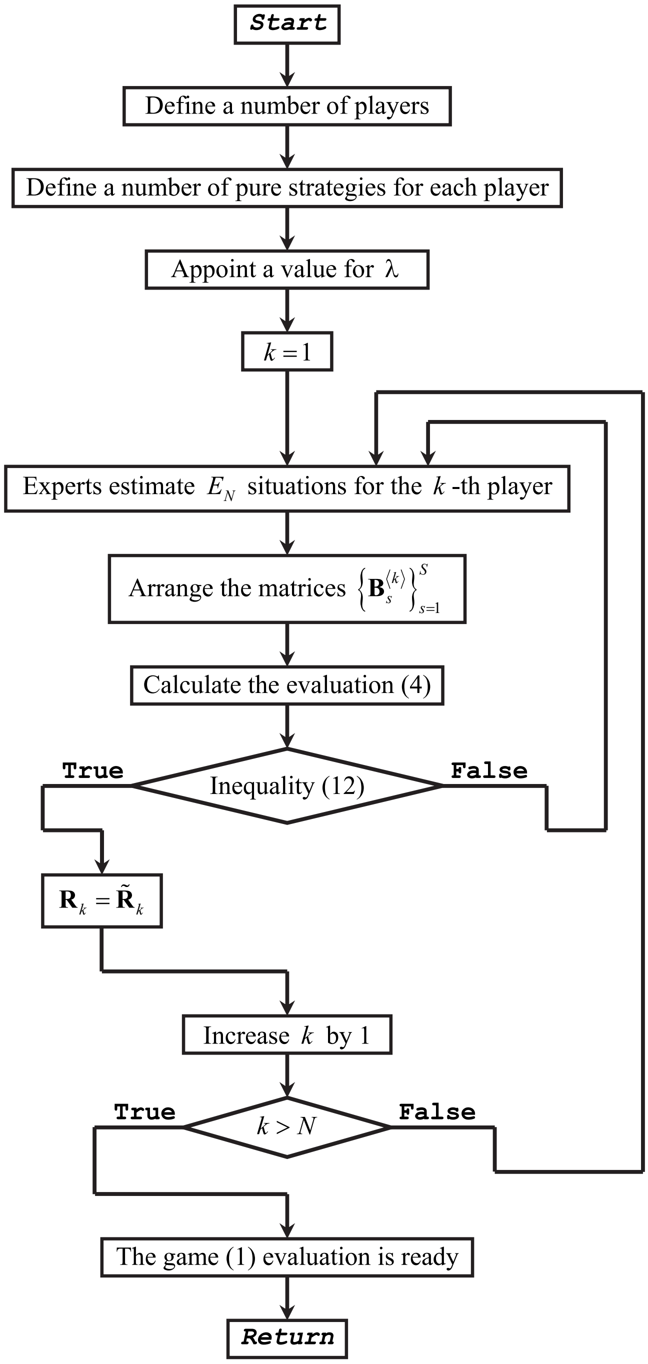


Fig. 1. A scheme for the method of processing expert estimations. The form of expert estimations is not specified. Arranging the matrices $\left\{\mathbf{B}_s^{\langle k\rangle}\right\}_{s=1}^{S}$ refers to putting experts' judgments into proper cells of, speaking generally, multidimensional (hyperparallelepipedic) matrices.

What should λ be appointed? In the worst case, $\left\{\mathbf{B}_s^{\langle k\rangle}\right\}_{s=1}^{S}$ are such that

$$\mathbf{B}_{s_1}^{\langle k\rangle} \neq \mathbf{B}_{s_2}^{\langle k\rangle} \quad \forall s_1 = \overline{1, S} \quad \text{and} \quad \forall s_2 = \overline{1, S} \quad \text{at} \quad s_1 \neq s_2 \tag{15}$$

by the greatest S. This is possible when $S = 2^{E_N}$. Then, obviously,

$$\tilde{\mathbf{R}}_k = \left(\tilde{r}_J^{\langle k\rangle}\right)_{\mathscr{F}} \quad \text{by} \quad \tilde{r}_J^{\langle k\rangle} = \frac{1}{2} \quad \forall J$$

for any set of indices J. If matrix $\mathbf{B}_s^{\langle k\rangle}$ has $E_N^{\langle 0\rangle}$ zeros and $E_N^{\langle 1\rangle}$ ones, the distance

$$\rho_{\tilde{\mathscr{B}}}\left(\mathbf{B}_s^{\langle k\rangle}, \tilde{\mathbf{R}}_k\right) = \sqrt{\left(0 - \frac{1}{2}\right)^2 \cdot E_N^{\langle 0\rangle} + \left(1 - \frac{1}{2}\right)^2 \cdot E_N^{\langle 1\rangle}} =$$

$$= \sqrt{\frac{E_N^{\langle 0\rangle}}{4} + \frac{E_N^{\langle 1\rangle}}{4}} = \frac{\sqrt{E_N}}{2}$$

and

$$\frac{1}{S}\sum_{s=1}^{S} \rho_{\tilde{\mathscr{B}}}\left(\mathbf{B}_s^{\langle k\rangle}, \tilde{\mathbf{R}}_k\right) = \frac{1}{S} \cdot \frac{\sqrt{E_N}}{2} \cdot S = \frac{\sqrt{E_N}}{2}.$$

Clearly, there must be $\lambda < \frac{\sqrt{E_N}}{2}$. Anyway, λ should be adjusted down off the value $\frac{\sqrt{E_N}}{2}$.

VII. Adjustment of λ

Adjustment of λ is really sensible when approximately the same experts successively evaluate a series of the players' payoff matrices or a series of games. This features Fig. 2, where

$$\tilde{\rho} = \frac{1}{S}\sum_{s=1}^{S} \rho_{\tilde{\mathscr{B}}}\left(\mathbf{B}_s^{\langle k\rangle}, \tilde{\mathbf{R}}_k\right) \tag{16}$$

and either

$$\lambda^{***} = \frac{\tilde{\rho} + \lambda^{**}}{2}, \tag{17}$$

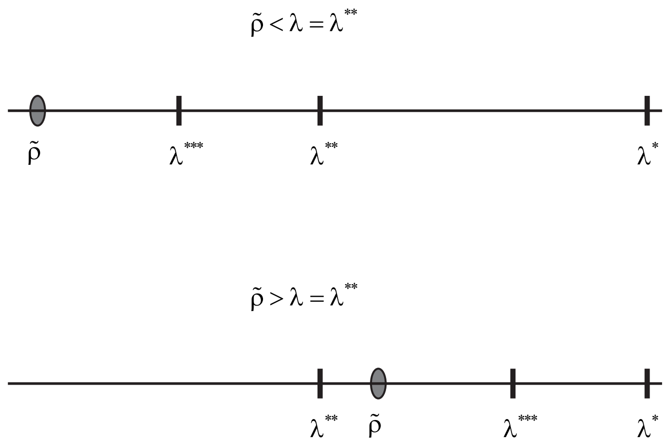
or

$$\lambda^{***} = \frac{\tilde{\rho} + \lambda^{*}}{2}, \tag{18}$$

with values

$$\left\{\lambda^{*}, \lambda^{**}, \lambda^{***}\right\},$$

of λ in the three successive estimation procedures. The adjusted value λ^{***} by (17) or (18) of λ refers to the third procedure. The current procedure is counted the second one, and its value λ^{**} was adjusted before to $\lambda^{**} < \lambda^{*}$ by the value λ^{*} of the previous procedure (which is counted the first one). In the third procedure (after the second procedure), the value of λ is either $\lambda^{***} < \lambda^{**}$ or $\lambda^{***} > \lambda^{**}$ depending on the value (16).



λ^{*} is a value of λ in the previous estimation procedure

λ^{**} is a value of λ in the current estimation procedure

λ^{***} is a value of λ in the next estimation procedure

point of the value of the left-side term of inequality (12)

Fig. 2. A sketch of how λ can be adjusted. Upper line part is for the case when inequality (12) turns true, and the lower one is valid when inequality (12) turns false. As it was substantiated before, value λ^{*} is reckoned to be less than $\frac{\sqrt{E_N}}{2}$. In particular, such an adjustment helps avoiding endlessness of loops in Fig. 1.

VIII. Application

Non-cooperative games are well-defined models of allocation (distribution/delivering/consuming) of limited or restricted resources. The game models have application in many fields concerning economics, engineering, politics, jurisprudence, pedagogics, etc. Real implementation of the game solution, which mostly is either Nash equilibrium strategies or Pareto efficiency strategies, is possible only when all the game payoffs are single-point evaluated. The evaluation, including experts' binary estimations and subsequent data processing, is implemented fast by embedding quiz tables in social (corporative) computer network pages. Such embedding does not require special skills or knowledge, so an ordinary user can do that.

IX. Discussion

The stated binary form evaluation procedure excludes payoffs equal 0.5 to $0.5 \pm \delta$. This is a demerit because middle-like payoffs or aftermaths do happen in real practice. Another demerit is uncertain λ which is adjusted.

Nonetheless, binary form evaluation procedure allows evaluating the whole game raw model in the most appropriate view, i.e., in the $[0;1]$-form implying $[0;1]$-valued payoff matrices. This also grants opportunity to compare effectiveness of situations when, for instance, the most efficient situation by Pareto is searched. The ultimate simplicity and no requirement for proficient and confident experts ensure high speed of binary form evaluation procedures.

X. Conclusion

This article considers a problem of obtaining evaluation of a non-cooperative game. The game evaluation implies getting just payoff matrices, when the players' pure strategy sets are pre-defined. Thus, this article proposes a method of obtaining the players' payoff matrices via expert estimations when they judge on each game situation (for every player) with 1 or 0. Proper attention should be devoted to relations amongst situations, but this is not required explicitly or compulsory. Explicit requirements (5) and (12) are provided for obtaining disambiguous payoff averages along with the clustered payoff matrices. The less λ the high density of the matrix cluster is meant to be. The number of experts to be appointed relates to the density. The greater λ the more experts are needed, but their confidences may be lower than for less λ. Some special cases with estimated λ are supposed to be considered in further investigation.

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