DE GRUYTER OPEN

Information Technology and Management Science

doi: 10.1515/itms-2014-0017

2014 / 17

Alternative Methods for Combining Probability Boxes

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Abstract **– Probability boxes (p-boxes) are used as a tool for modeling uncertainty regarding probability distributions in the sets of relevant elements (random events, values of the random variable etc.). To combine information produced by two or more p-boxes, Dempster's rule for belief combination is commonly used. However, there are plenty of other rules for belief combination developed within the theory of evidence. The purpose of this paper is to present and analyze some widespread rules of that kind as well as examine their potentialities regarding combining the information provided by probability boxes.**

Keywords **– Basic probability assignments, belief combination rules, *p*-boxes, theory of evidence.**

I. Introduction

Probabilistic evaluations represent occurrence chances of random events. Whenever initial data are missing or insufficient, the evaluation of relevant probabilities becomes quite difficult. If the evaluation is made by experts, the validity of the estimates obtained in principle cannot be evaluated a priori. To enable consideration of uncertainties related to probability evaluation, different techniques can be used. Common idea behind all those techniques is that instead of a single probability function, boundary probabilities are specified. It is supposed that a true probability function is between those boundary probability functions. However, situations are frequently possible when the accumulated distribution functions can only be constructed on different sets of relevant elements. Situations of this kind occur when analyzing risks and/or safety of technical system operation. Among the reasons causing this kind of uncertainties, the following can be mentioned [2]:

- imprecisely defined probability distributions;
- ill-identifiable or even unknown correlations;
- essential measurement errors;
- impact of unrecognized factors on the model output;
- small sizes of samples;
- uncertainty of a model;
- non-stationarity (inconstant distributions).

To model this kind of uncertainties, the *p*-boxes technique was suggested. Theoretical foundations of the technique were first described in [1], [2]. The use of representation exploiting probability boxes in the structural analysis based on the method of finite elements is discussed in [8], whereas *p*-box application in the general context of risk assessment is examined in [3].

In what follows, to make the presentation simple and visual, it will be assumed that X is a set of real numbers, $\boldsymbol{R}$. This assumption in no extent will affect the commonness of material presentation.

The main idea of the p-box techniques is as follows. If, due to some reasons, it is not possible to construct a single probability distribution function on $\boldsymbol{R}$, boundary distribution functions similar to those depicted in Fig. 1 are constructed.

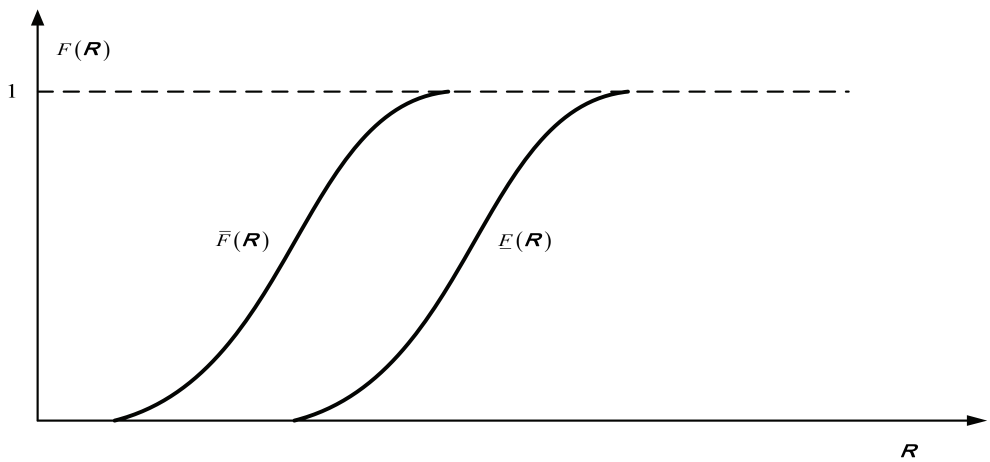

Fig. 1. Boundary probability distribution functions in the set of relevant values, $\boldsymbol{R}$.

It is stated that a real unknown distribution function $F(\boldsymbol{R})$ is in between those boundary functions. $\underline{F}(R)$ is the left boundary of the set of possible distribution functions, which represents the values of the accumulated distribution function for $\boldsymbol{R} < r$. $\overline{F}(R)$ is the upper boundary of possible distribution functions. If a set of the lower probability values on $\boldsymbol{R}$, $\underline{P}(R)$ is known, the boundary distribution functions can be expressed as follows:

$$\overline{F}(r)=1-\underline{P}\left(R>r\right); \qquad (1)$$

$$\underline{F}\left(r\right)=\underline{P}\left(R\le r\right). \qquad (2)$$

From Fig. 1 it follows that the left boundary $\overline{F}(R)$ is the upper boundary for distribution function values and the lower boundary for value R . Instead, the right-hand boundary $\underline{F}(R)$ is the lower boundary for the values of distribution function and the upper boundary for value R .

The construction of probability boxes can be both parametric and non-parametric. The non-parametric construction is based on the assumption that the form and parameters of underlying probability distribution are unknown. The boundaries of the constructed p-box can have a deliberate form.

How could information provided by two or more probability boxes be aggregated? If boundary functions of distributions for combined probability boxes have a continuous form, such boxes should be discretized in advance. There are two techniques of probability box discretization. The essence of the first one, the so-called boundary discretization, is depicted in Fig. 2.

The second technique of discretization called discretization over mean points is illustrated in Fig. 3.

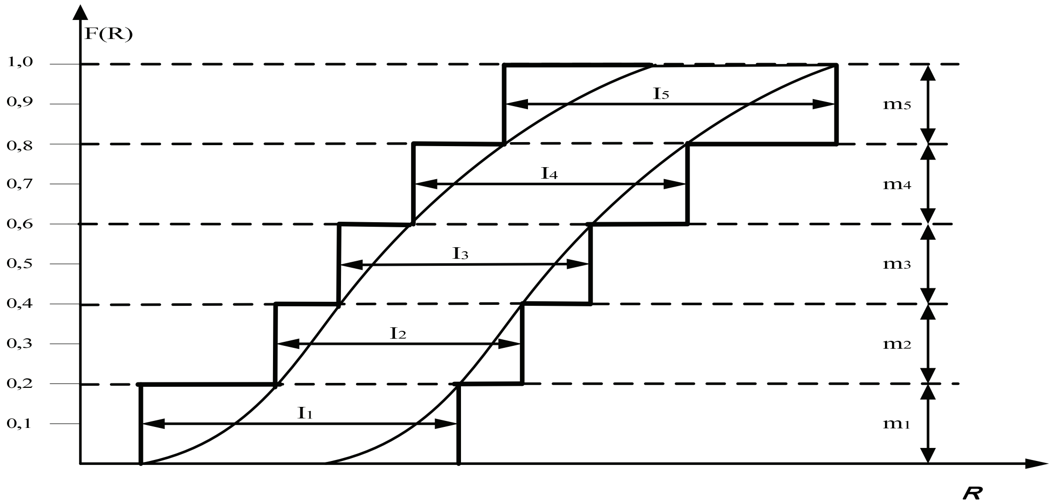

Fig. 2. Schematic representation of the method of boundary discretization of probability boxes.

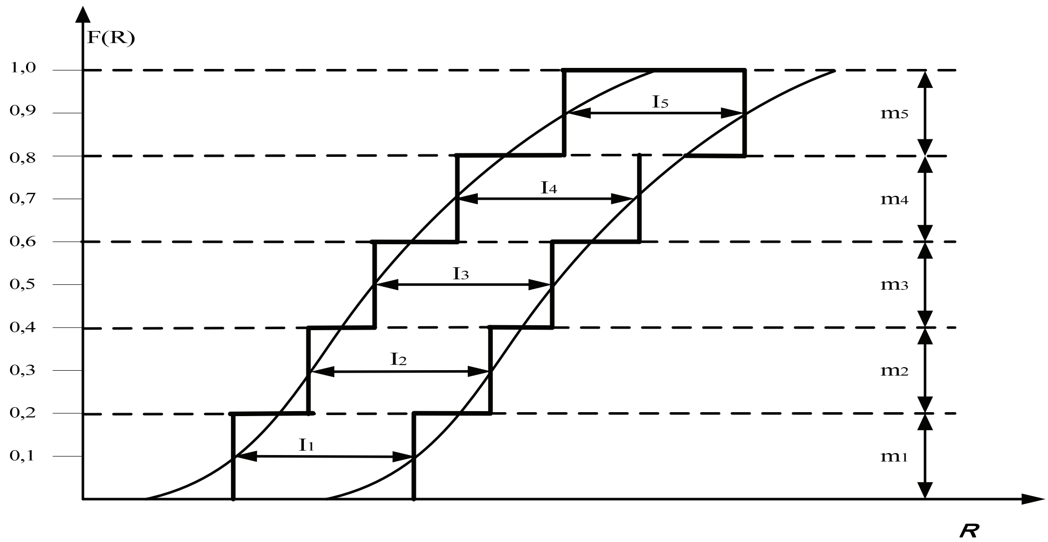

Fig. 3. Schematic representation of probability box discretization method based on mean points.

Let us assume that we have two stepwise or discretized probability boxes constructed on the basis of two independent information sources. The task is to aggregate information from those sources. The next section considers basics of the theory of evidence and Dempster's belief combination rule that makes it possible to solve the task in a standard way.

II. Basics of the Theory of Evidence

The basics of the theory of evidence (Dempster-Shafer theory) were first described in [4]. A more detailed description of the theory can be found in Chapters 2 and 3 of [8]. Below, short data about the basic concepts and definitions of this theory are outlined, which are necessary for the explication of procedures for combining probability boxes.

One of the fundamental concepts of the theory is the concept of **frame of discernment** $\Omega = \{\omega_i / i = 1,...,n\}$. It is formed of the elements under consideration. The notion of elements can be treated quite widely depending on the context. Elements can be possible values of unknown variable, individuals who are suspected of making a crime, certain events etc. Only one element is true. Let us denote it as ω_0 and call it a real world.

The essence of Dempster-Shafer's theory is as follows. Based on the evidences available, subsets of elements $A \subseteq \Omega$ are determined; the subsets may contain a real world ω_0. If a real value of some unknown variable is under consideration, the role of subsets will be played by respective intervals. Relevant subsets or intervals are called ***focal elements***. Function $m : 2^{\Omega} \to [0,1]$ that is called ***basic probability assignment*** can be correlated with the frame of discernment; the function satisfies these requirements:

$$m(\varnothing) = 0;$$

$$\sum_{A \subseteq \Omega} m(A) = 1 \text{ for all } A \subseteq \Omega.$$

The kernel of the theory of evidence is the concept of ***belief function*** $bel : 2^{\Omega} \to [0,1]$ that meets the following requirements:

$$bel(\varnothing) = 0;\ bel(\Omega) = 1;$$

For every integer n and every set $A_1,...,A_n$ of subsets Ω we have

$$bel(A_1 \cup ... \cup A_n) = bel(A_i) - \sum_{i<j} bel(A_i \cap A_j) + ... + (-1)^{n+1} bel(A_1 \cap ... \cap A_n).$$

Belief functions are correlated with basic probability assignments in this way:

$$bel(A) = \sum_{B_i \subset A} m(B_i). \quad (3)$$

An important question in the Dempster-Shafer theory is the combination of beliefs produced by different evidences. Most widespread rule of belief combination is Dempster's rule [5]. Let on the basis of the first group of evidences there be assigned basic probability masses to certain subsets (focal elements) on the frame of discernment $A_i \subset \Omega, i = 1,...,m$. Assume that there is another group of evidences on whose basis there are assigned basic masses of probability to subsets $B_j \subset \Omega, j = 1,...,n$. The combined mass of probability corresponding to the overlapping of focal elements $A_i \cap B_j$ is expressed as follows:

$$m_{12}(A_i \cap B_j) = m_1\{A_i\} m_2\{B_j\}. \quad (4)$$

For any subset C, comprising any number of subsets $A_i \cap B_j$, the combined mass of probability can be calculated as a sum

$$\sum_{\substack{i,j \\ A_i \cap B_j \subset C}} m_1\{A_i\} m_2\{B_j\}. \quad (5)$$

Dempster's rule of combination foresees the normalization of the combined masses of probability corresponding to non-empty intersections of marginal focal elements. The value of the normalizing constant K is calculated by this expression:

$$K = \left[1 - \sum_{\substack{i,j \\ A_i \cap B_j \neq \varnothing}} m_1\{A_i\} m_2\{B_j\} \right]^{-1}. \quad (6)$$

Let us consider an example illustrating the combination of probability boxes on the basis of Dempster's rule (the idea is borrowed from [8]).

Example 1. Fig. 4 shows three probability boxes constructed on the basis of evaluations of experts A, B and C in the set of values of random variable π.

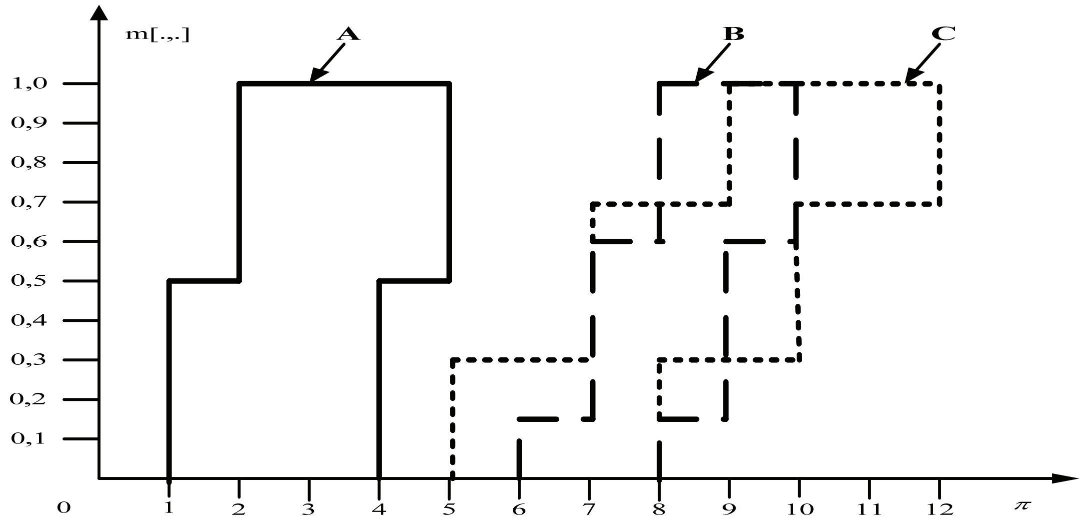

Fig. 4. Probability boxes constructed by three experts A, B and C.

Let us express the information represented by these boxes in the numeric form.

Expert A:	Expert B:	Expert C:
[1, 4] $m_1 = 0.5$;	[6, 8] $m_2 = 0.2$;	[5, 8] $m_3 = 0.3$;
[2, 5] $m_1 = 0.5$.	[7, 9] $m_2 = 0.4$;	[7, 10] $m_3 = 0.4$;
	[8, 10] $m_2 = 0.4$.	[9, 12] $m_3 = 0.3$.

The figures in square brackets represent intervals of probability boxes but the values m(.) are basic probability assignments related to the corresponding intervals.

It is obvious that the information provided by probability box A cannot be combined with the information provided by probability boxes B and C because Dempster's rule can only be used for overlapping probability boxes.

By combining basic probability assignments for the overlapping intervals of probability boxes B and C according to expression (5) and normalizing the results by expression (6) we get:

m_{BC} [6, 8] = 0.0857; m_{BC} [7, 8] = 0.2857; m_{BC} [7, 9] = 0.2286; m_{BC} [8, 10] = 0.2286;
m_{BC} [9, 10] = 0.1714.

The combined probability box is shown in Fig. 5. As can be seen in Fig. 5, the resulting probability box is narrower than the initial boxes. The reason for that is that Dempster's rule of combination takes into account only the overlapping parts of intervals of the initial probability boxes.

Dempster's rule of combination has a very strong underlying logical basis and is characterized by high conservatism. The result of combination only depends on the information, which is provided by both sources. The information provided by a separate source is not included in the result of combination and serves only for combination result normalization.

The shortcoming of Dempster's rule of combination is that it can produce unnatural results in certain specific conditions [6], [7].

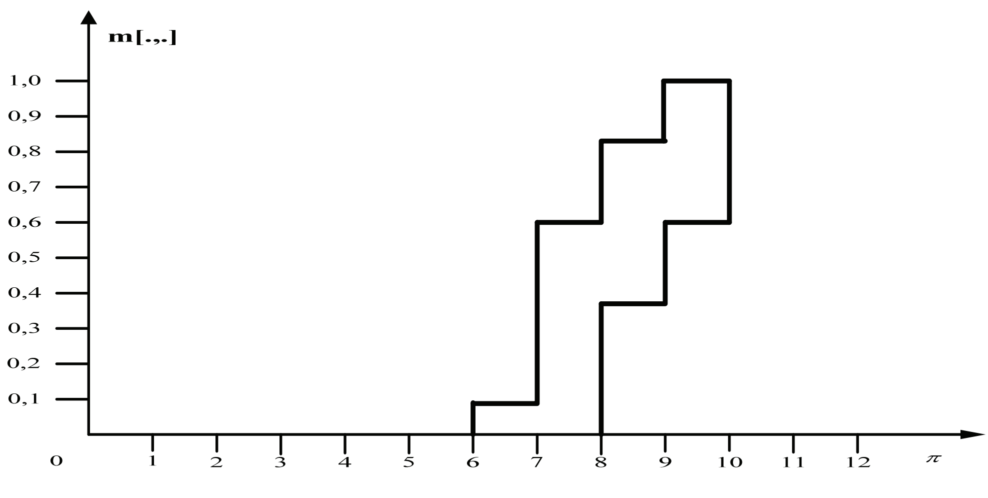

Fig. 5. Probability box obtained through the combination of probability boxes B and C according to Dempster's rule (see Fig. 4).

III. Alternative Methods for the Combination of Probability Boxes

Nowadays, a large number of alternative rules for belief combination exist. More details on this kind of techniques can be found in [9]). This section considers most famous of them. One of the rules of this kind is Yager's rule of combination [10]. Yager has refused from the idea to ascribe all combined masses of probability related to empty overlapping of marginal focal elements to an empty set. Like in Dempster's rule, the value of each combined probability mass is calculated here as an orthogonal sum of the corresponding basic masses of probability (5). The normalization of the results is not foreseen. Though Yager's rule allows one to get rid of the main shortcoming of Dempster's rule; it possesses another shortcoming. It is impossible to construct a probability box on the basis of the results of combination because the sum of resulting masses is not equal to 1. Due to that reason, Yager's rule cannot be employed for combining probability boxes.

Dempster's rule of combination assumes as a basis overlapping marginal focal elements that are determined on the basis of different groups of evidences. This rule, however, does not consider the extent of such overlapping, whereas an alternative rule of combination, Zhang's rule [12], does take into account the extents of overlapping of the corresponding focal elements. If a subset C is the result of overlapping of subsets A and B, $C = A \cap B$, Zhang introduces the evaluation of the extent of overlapping of these subsets in this manner:

$$r(A,B) = \frac{|C|}{|A||B|} = \frac{|A \cap B|}{|A||B|}, \qquad (7)$$

where $|A|$, $|B|$, $|A \cap B|$ are cardinalities of subsets A, B, $A \cap B$, respectively.

Evaluations (7) are calculated for each pair of the overlapping marginal focal elements. The value of the combined mass of probability related to a certain subset $C \subset \Omega$ is calculated as follows:

$$m(C) = K \sum_{A \cap B = C} \left[\frac{|C|}{|A||B|} m_1(A) m_2(B) \right], \qquad (8)$$

where K is a normalizing constant.

Zhang's rule of combination also takes into account only the overlapping parts of intervals of probability boxes. The results of combination according to that rule slightly differ from those obtained using Dempster's rule. That difference is due to the principles of combination underlying both rules.

Zhang's rule can be recommended for the combination of probability boxes in cases when due to some reasons it is necessary to take into account the extent of overlapping of initial boxes.

There are also rules of combination developed that are based on one or another kind of averaging of the initial masses of probability. One of widespread rules of this kind is called ***ρ-averaging.*** The idea of that averaging is quite simple. If, based on the n groups of evidences basic probability masses $m_i(A)$, $i = 1, \dots, n$, are assigned to a subset $A \subset \Omega$, then the combined mass of probability ascribed to that subset is calculated by expression

$$m_{1,\dots,n}(A) = \frac{1}{n} \sum_{i=1}^{n} w_i m_i(A), \qquad (9)$$

where w_i is a coefficient (weight) characterizing the extent of reliability of the i-th group of evidences. In particular case the averaging can be made without introducing coefficients (assuming the values of all w_i equal 1).

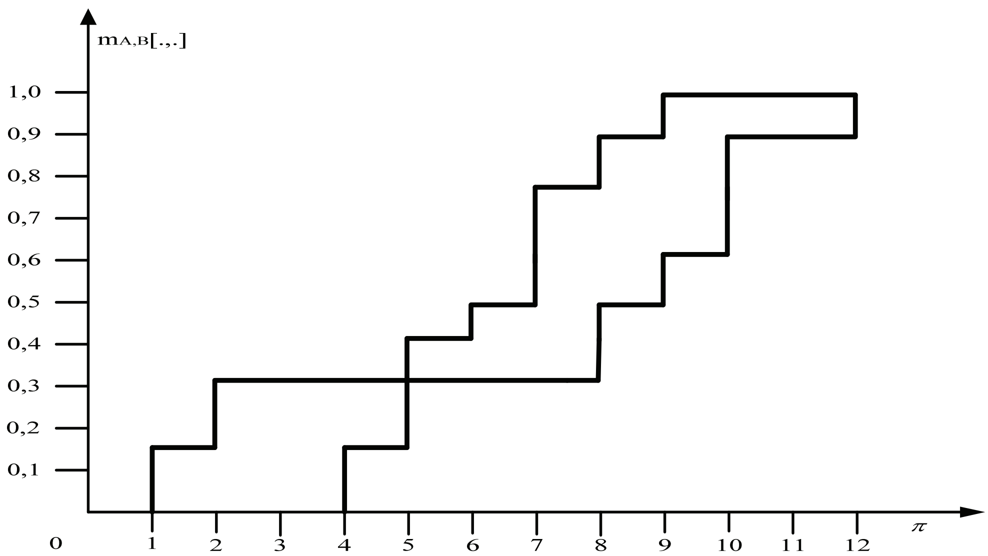

Fig. 6. A probability box obtained as a result of combination of three initial probability boxes in Example 1 on the basis of the rule of ρ-averaging.

Applying the rule of ρ-averaging to all initial probability boxes in Example 1, we get the following result of combination:

$m_{A,B,C}[1, 4] = 1/3*0.5 = 0.167$; $m_{A,B,C}[2, 5] = 1/3*0.5 = 0.167$; $m_{A,B,C}[5, 8] = 1/3*0.3 = 0,100$;

$m_{A,B,C}[6, 8] = 1/3*0.2 = 0.067$; $m_{A,B,C}[7, 9] = 1/3*0.4 = 0.133$; $m_{A,B,C}[7, 10] = 1/3*0.4 = 0.133$;

$m_{A,B,C}[8, 10] = 1/3*0.4 = 0.133$;

$m_{A,B,C}[9, 12] = 1/3*0.3 = 0.100$.

The resulting probability box is shown in Fig. 6.

It is clear that the rule of ρ-averaging allows combining non-overlapping probability boxes, which might be quite important in certain specific cases.

As opposed to the rule of ρ-averaging that averages probability masses by initial intervals, under ***x-convolving averaging*** specific combination of the initial relevant intervals is performed. The boundaries of each resulting interval are determined as the mean values of the boundaries of the initial intervals.

The combined values of probability masses ascribed to each resulting interval are calculated in a standard manner as the multiplications of the marginal probability masses assigned to the initial intervals.

An essential feature of that rule of combination and also of the rule of ρ-averaging is that it forms and takes into account the resulting intervals even in the cases when the initial intervals do not overlap.

Applying the rule of ***x-convolving averaging*** to probability boxes A and B in Example 1, we get the following results of combination:

$m_{AB}[3,5, 6] = 0.10$; $m_{AB}[4, 6,5] = 0.30$;
$m_{AB}[4,5, 7] = 0.40$; $m_{AB}[5, 7,5] = 0.20$.

The resulting probability box is shown in Fig. 7.

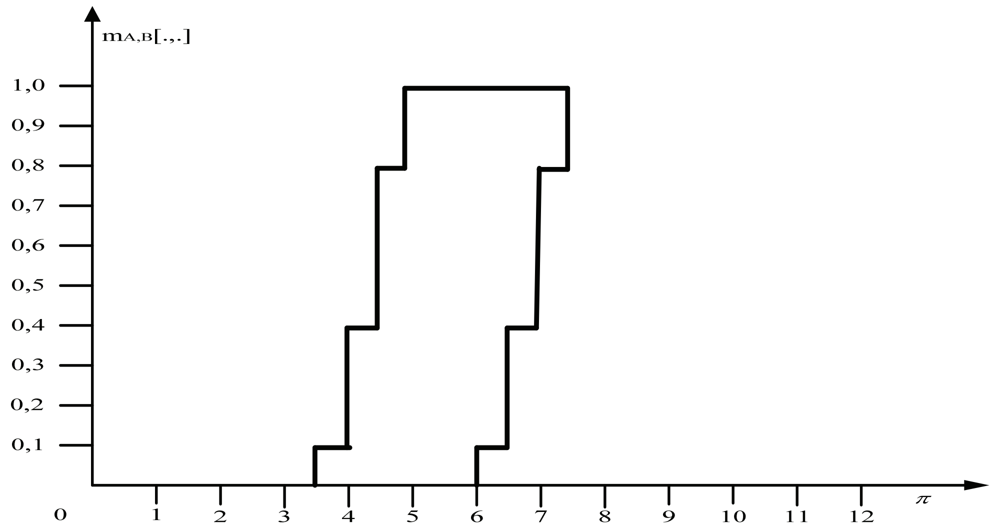

Fig. 7. Probability box obtained as a result of combining initial probability boxes A and B in Example 1 on the basis of the rule of x –convolving averaging.

A specific rule of belief combination is the disjunctive rule of Dubois and Prade. That rule is similar to the rule of ***x-convolving averaging***, the only difference being in the determination of the boundaries of the resulting intervals. In the rule of ***x-convolving averaging***, each of the boundaries is determined as the mean of the corresponding boundaries of the initial intervals. In the given rule, the lower boundary of the resulting interval is equal to the minimal value of the lower boundaries of the initial intervals while the upper boundary is equal to the maximum value of the upper boundaries of the initial intervals.

Let us combine the initial probability boxes A and B in Example 1 by the disjunctive rule of Dubois and Prade. As a result, we have

$m_{AB}[1,8] = 0.10$; $m_{AB}[1,9] = 0.20$; $m_{AB}[1,10] = 0.20$;
$m_{AB}[2,8] = 0.10$; $m_{AB}[2,9] = 0.20$; $m_{AB}[2,10] = 0.20$.

The resulting probability box is depicted in Fig. 8.

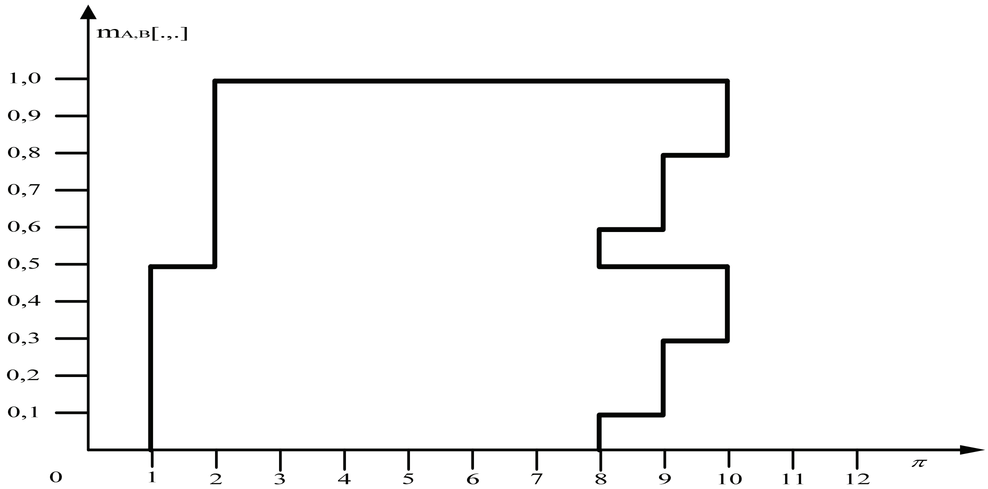

Fig. 8. A probability box obtained as a result of combining initial probability boxes A and B in Example 1 on the basis of the Dubois and Prade disjunctive rule of combination.

IV. Conclusion

When developing the theory of probability boxes for combining information provided by two or more probability boxes, the use of Dempster's rule of belief combination was suggested. This suggestion is probably based on the wide reputation and applicability of rule. However, plenty of other rules for combining beliefs are proposed. Most widespread rules of this kind are briefly described in this paper.

All the above-mentioned rules of belief combination except for Yager's rule can be in principle used for combining probability boxes.

It should be noted that all those rules can be divided into three groups: (1) rules working only on the overlapping of intervals that form probability boxes; (2) rules based on the averaging of initial probability masses for initial intervals and (3) rules based on the specific formation of resulting intervals on the basis of the initial intervals. Combination rules of Dempster and Zhang fall to the first group. Zhang's rule only differs from Dempster's rule in that it takes into account the extent of overlapping of the initial intervals when calculating resulting probability masses. Both these rules can be used in the cases when a high confidence of the resulting evaluations is required. In general case, preference has to be given to Dempster's rule as it has a more general character and is simpler from the computational point of view.

The rule of ρ-averaging is ascribed to the second group and has a universal character. It takes into account all initial probability masses for relevant intervals. The advantage of the method is the possibility of accounting the extent of confidence for different groups of evidences (initial probability boxes).

The next two methods are labeled to the third group. An essential feature of the rule of x-convolving averaging is the specific creation of the resulting intervals, which are the result of averaging of the initial intervals. However, under such a combination, the uncertainty of the resulting evaluations may be higher than that of the initial evaluations, which essentially impedes the interpretation of the results obtained and makes the deduction of validated conclusions quite problematic. That is why the given rule can be recommended for practical use only in those cases, when it is desired to use considerable amount of the initial information not taking into account its potentially contradictory character.

These conclusions are even more relevant for the Dubois and Prade disjunctive rule of combination. The advantage of the rule is that it makes use of the whole initial information. The shortcoming of the method is a high uncertainty of the results. Due to that, the rule can only be used in exceptional cases.

A general conclusion that can be made on the basis of this paper is – when combining probability boxes researchers should not restrict themselves to Dempster's rule of combination; instead, alternative rules of combination have to be widely used. The choice of proper rule of combination has to be dictated by conditions of a specific task and specific requirements to the results.

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Olegs Užga-Rebrovs, Galina Kuļešova. Alternatīvās metodes varbūtības kastu kombinēšanai

Liela daļa lietišķo uzdevumu varbūtības lielumu novērtēšanai tiek veikta uz ekspertu subjektīvo spriedumu pamata. Tā kā ekspertiem bieži ir grūtības ar viennozīmīgu relevanto varbūtību novērtējumu, tiek izstrādātas pieejas, kuras ļauj modelēt papildu nenoteiktības, kas ir saistītas ar ekspertu varbūtības novērtējumiem. Tāda veida zināmās pieejas ir nenoteiktās varbūtības un izplūdušās varbūtības. Ar varbūtību kastu palīdzību tiek modelētas nenoteiktības, kas ir saistītas ar varbūtību sadalījuma ekspertu novērtējumu. Ja eksperts nav pārliecināts, ka novērtējama sadalījums ir ticams, viena varbūtību sadalījuma vietā viņš uzdod divus varbūtību sadalījumus, kuri veido varbūtību kasti. Varbūtību sadalījumiem, kuri veido varbūtību kasti, var būt dažāda forma. Sadalījumi var būt arī nesimetriski. Problēmas rodas tad, kad jākombinē informācija, kas tiek piedāvāta ar divām vai vairākām varbūtību kastēm, kuras konstruēja neatkarīgi eksperti. Oriģinālos darbos par varbūtību kastēm šim mērķim tiek piedāvāts izmantot Dempstera pārliecību kombinēšanas likumu. Metodes trūkums − nepieciešamība normēt rezultējošās pārliecības. Šī raksta mērķis ir parādīt iespējas izmantot alternatīvas metodes informācijas kombinēšanai, kura tiek iegūta no divām vai vairākām varbūtības kastēm. Šīs metodes var pielietot tieši, ja varbūtību koki tiek ierobežoti ar diskrētiem varbūtību sadalījumiem. Nepārtraukto varbūtību sadalījumu gadījumā sākotnējiem sadalījumiem jābūt diskretizētiem pirms kombinēšanas operāciju izpildes. Darbā ir minētas divas varbūtības koku diskretizācijas metodes: robeždiskretizācija un diskretizācija vidējos punktos. Varbūtības koku kombinēšanas alternatīvo metožu esamība ļauj izvēlēties piemērotāko metodi konkrēta uzdevuma kontekstā.

Олег Ужга-Ребров, Галина Кулешова. Альтернативные методы комбинирования вероятностных ящиков

Назначение вероятностных оценок в большом числе прикладных задач производится на основе субъективных мнений экспертов. Поскольку эксперты часто затрудняются дать однозначные оценки для релевантных вероятностей, разработаны подходы, позволяющие моделировать дополнительные неопределённости, связанные с экспертным назначением вероятностей. Наиболее известными подходами такого рода являются неопределённые вероятности и нечёткие вероятности. С помощью вероятностных ящиков моделируются неопределённости, связанные с экспертным конструированием вероятностных распределений. Если эксперт не уверен в достоверности оцениваемого вероятностного распределения, вместо одного вероятностного распределения он задаёт два граничных вероятностных распределения, которые и образуют вероятностный ящик. Вероятностные распределения, образующие ящик, могут иметь самую разнообразную форму и не быть симметричными. Проблемы возникают в том случае, когда нужно скомбинировать информацию, даваемую двумя или более вероятностными ящиками, сконструированными независимыми экспертами. В оригинальных работах по вероятностным ящикам для этой цели предлагается использовать правило комбинирования уверенностей Демпстера. Недостатком этого метода следует признать необходимость нормирования результирующих уверенностей. Цель настоящей статьи – показать возможности применения альтернативных методов комбинирования уверенностей для комбинирования информации, даваемой двумя или более вероятностными ящиками. Эти методы непосредственно применимы в случаях, когда вероятностные ящики ограничены дискретными вероятностными распределениями. В случае непрерывных вероятностных распределений эти распределения должны быть дискретизированы до выполнения операций комбинирования. В работе представлены два метода дискретизации вероятностных ящиков: метод граничной дискретизации и метод дискретизации на средних точках. Наличие альтернативных методов комбинирования вероятностных ящиков позволяет выбрать наиболее подходящий метод в контексте конкретной задачи.