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Optimization of the Fuzzy Investment Portfolio during the Time Period

Inna Sydoruk[1], Yuri Zaychenko[2], [1, 2] *National Technical University of Ukraine "Kiev Polytechnic Institute"*

Abstract – **The problem of portfolio optimization under uncertainty is considered. For its solution the application of fuzzy sets theory is suggested. Fuzzy portfolio optimization problem during the time period is stated, its model is provided, investigated, and algorithm of its solution presented. This problem includes two main criteria – portfolio profitableness and risk. A mathematical model of this problem is constructed and explored. For better estimation of stock profitableness, Fuzzy Group Method of Data Handling (FGMDH) is suggested. The experimental investigations of the suggested approach are carried out. The results with optimal portfolios based on forecasted profitableness are presented and its efficiency is evaluated.**

Keywords – **Forecasting, fuzzy portfolio, FGMDH, portfolio optimization, profitableness, stock prices.**

I. Introduction

Portfolio is a purposefully formed set of investment assets (real or financial investments) owned by an individual or a legal person. The aim of this set is to implement the pre-developed strategy and to achieve the investment objectives. The main objective of portfolio investment is to create for a set of investment assets such investment conditions that are inaccessible from the position of a single asset and possible only when it is combined with the other. This includes achieving the optimal combination for investors of such investment characteristics as profitableness and risk level.

Portfolio analysis exists, perhaps, as long, as people think about making rational decisions connected with the use of limited resources. However, the occurrence moment of portfolio analysis started with a publication of pioneer work by Harry Markovitz (Markovitz's Portfolio Selection) in 1952. The model offered in that work, simple enough in essence, has allowed catching the basic features of the financial market, from the point of view of the investor, and has supplied the last with the tool for development of rational investment decisions.

The central problem in the Markovitz's theory is the portfolio choice, i. e., a set of operations. Thus, in the estimation, both separate operations and their portfolios are considered: profitableness and risk of operations and their portfolios. The risk thus receives a quantitative estimation. The account of mutual correlation dependences between profitableness of operations appears the essential moment in the theory. This account allows making effective diversification of portfolio, leading to essential decrease in risk of a portfolio in comparison with risk of the operations included in it. At last, the quantitative characteristic of the basic investment characteristics allows defining and solving a problem of a choice of an optimum portfolio in the form of a problem of quadratic optimization.

However, the worldwide market crises during the last 20 years have shown that existing theories of optimization of share portfolios and forecasting of share indices have exhausted themselves, and essential revision of share management methods is necessary.

Thus, in the light of obvious insufficiency of available scientific methods for management of financial assets, the development of a fundamentally new theory of management of the financial system functioning in the conditions of essential uncertainty is needed. The great assistance to this theory was rendered by the theory of the fuzzy sets, which have been developed about half a century ago in fundamental works of Lotfi Zadeh.

In previous studies, we have discovered the application of classical probabilistic method and fuzzy set theory. We have considered direct, dual and multi-criteria optimization portfolio problems.

The purpose of the present article is the research and experimental analysis of direct optimization problem during the time period.

II. Problem Statement

Let us consider a share portfolio from N components and its expected behavior at time interval $[0,T]$. Each portfolio component is characterized by the financial profitableness r_i, $I = 1,N$.

The holder of a share portfolio – the private investor, the investment company, mutual fund – operates the investments, being guided by certain reasons. On the one hand, the investor tries to maximize the profitableness. On the other hand, it fixes a maximum permissible risk of an inefficiency of the investments. We will assume the capital of the investor be equal to 1. The problem of optimization of a share portfolio consists in the finding of a vector of share price distribution of papers in a portfolio $x = \{x_i\}$, $i = \overline{1,N}$ of the investor maximizing the income at the set risk level (obviously, that $\sum_{i-1}^{N} x_i = 1$).

Weaknesses of a Classical Markovitz's Model

In the process of practical application of Markovitz's model, its drawbacks were found out:

1. The hypothesis about normality profitableness distributions in practice does not prove to be true.
2. Stationarity of price processes also not always is in practice.
3. At last, the risk of actives is considered a dispersion standard deviation of the prices of securities from the expected value, i.e., a decrease in profitableness of securities in relation to the expected value, and profitableness increase in relation to an average are estimated absolutely the same.

Though for the proprietor of securities these events are absolutely different.

These weaknesses of Markovitz's theory define the necessity of the development of essentially new approach of definition of an optimum investment portfolio.

III. Fuzzy Set Portfolio Model

Main principles and idea of the method are as follows:

- The risk of a portfolio is not its volatility, but possibility that expected profitableness of a portfolio will appear below some pre-established planned value.
- Correlation of assets in a portfolio is not considered and not accounted.
- Profitableness of each asset is not a random fuzzy number. Similarly, restriction on an extremely low level of profitableness can be both usual scalar and fuzzy number of any kind. Therefore, optimization of a portfolio in such a statement may mean in that specific case the requirement to maximize expected profitableness of a portfolio at a point of time T at the fixed risk level of a portfolio.
- Profitableness of a security on termination of ownership term is expected to be equal to r and is in a settlement range.

Denote for the *i*-th security:

$\tilde{r}_i$ – expected profitableness of *i*-th security;

r_{i1} – the lower border of profitableness of *i*-th security;

r_{i2} – the upper border of profitableness of *i*-th security.

$r_i = (r_{i1}, \tilde{r}_i, r_{i2})$ – profitableness of *i*-th security, is a triangular fuzzy number.

Then profitableness of a portfolio:

$$r = \left(r_{\min} = \sum_{i=1}^{N} x_i r_{i1}; \tilde{r} = \sum_{i=1}^{N} x_i \tilde{r}_i; r_{\max} = \sum_{i=1}^{N} x_i r_{i2} \right), \quad (1)$$

where x_i is a portion of *i*-th asset in portfolio, and

$$\sum_{i=1}^{N} x_i = 1, x_i \geq 0, i = \overline{1, N} \quad (2)$$

Critical level of profitableness of a portfolio at the moment of T is r^*.

IV. Mathematical Model of a Fuzzy Optimization Problem

To define the structure of a portfolio, which will provide the maximum profitableness at the set risk level, it is required to solve the following problem (1) – (6):

$$\{x_{opt}\} = \{x\} \mid r \to \max, \quad \beta = const, \quad (3)$$

where r is profitableness, β is a desired risk, vector's components x satisfy (2).

As shown in (1) – (6), the most expected value risk degree of a portfolio is defined:

$$\beta = \begin{cases} 0, & if \quad r^* < r_{\min} \\ R\left(1 + \frac{1-\alpha_1}{\alpha_1}\ln(1-a_1)\right), & if \quad r_{\min} \leq r^* \leq \tilde{r} \\ 1-(1-R)\left(1 + \frac{1-\alpha_1}{\alpha_1}\ln(1-a_1)\right), & if \quad \tilde{r} \leq r^* < r_{\max} \\ 1, & if \quad r^* \geq r_{\max} \end{cases} \quad (4)$$

where

$$R = \begin{cases} \frac{r^* - r_{\min}}{r_{\max} - r_{\min}}, & if \quad r^* < r_{\max} \\ 1, & if \quad r^* \geq \mathrm{r}_{\max} \end{cases} \quad (5)$$

$$\alpha_1 = \begin{cases} 0, & if \quad r^* < r_{\min} \\ \frac{r^* - r_{\min}}{\tilde{r} - r_{\min}}, & if \quad r_{\min} \leq r^* < \tilde{r} \\ 1, & if \quad r^* = \tilde{r} \\ \frac{r_{\max} - r^*}{r_{\max} - \tilde{r}}, & if \quad \tilde{r} < r^* < r_{\max} \\ 0, & if \quad r^* \geq r_{\max} \end{cases} \quad (6)$$

The profitableness of a portfolio is shown in (1).

V. Mathematical Model of Portfolio Optimization during the Period

In this case we must define the structure of the portfolio, which provides the maximum average return for a given level of risk. Thus, we calculate the profitableness from (3) as:

$$\tilde{r} = \frac{1}{T}\sum_{t=1}^{T}\sum_{i=1}^{N} x_{ti}\tilde{r}_{ti},$$

where $\tilde{r}_{ti}$ – the expected profitableness of *i*-th security in time unit t, T – the number of time units. We should find an optimal portfolio from the following problem:

$$\tilde{r} = \sum_{i=1}^{N} x_i \tilde{r}_i \to \max, \quad (7)$$

$$\beta = const, \quad (8)$$

$$\sum_{i=1}^{N} x_i = 1,\ x_i \geq 0, i = \overline{1,N} \quad (9)$$

At a risk level variation β 3 cases are possible. We will consider in detail each of them.

1. $\beta = 0$

From (4) it is evident that this case is possible when $r^* < \frac{1}{T}\sum_{t=1}^{T}\sum_{i=1}^{N} x_{ti} r_{ti1}$

We receive the following problem of linear programming (9) – (11):

$$\tilde{r} = \frac{1}{T}\sum_{t=1}^{T}\sum_{i=1}^{N} x_{ti}\tilde{r}_{ti} \to \max, \quad (10)$$

$$r^* < \frac{1}{T}\sum_{t=1}^{T}\sum_{i=1}^{N} x_{ti} r_{ti1}, \quad (11)$$

$$\sum_{i=1}^{N} x_i = 1, x_i \geq 0, i = \overline{1,N}, t = \overline{1,T} \quad (12)$$

The found result of the problem decision (10) – (12) vector $x = \{x_i\}$ $i = \overline{1,N}$ is a required structure of an optimum portfolio for the given risk level.

2. $\beta = 1$

From (4) it follows that this case is possible when

$$r^* \geq \frac{1}{T}\sum_{i=1}^{T}\sum_{i=1}^{N} x_{ti} r_{ti2}.$$

We receive the following problem of linear programming (13) – (15):

$$\tilde{r} = \frac{1}{T}\sum_{t=1}^{T}\sum_{i=1}^{N} x_{ti}\tilde{r}_{ti} \to \max, \quad (13)$$

$$r^* \geq \frac{1}{T}\sum_{i=1}^{T}\sum_{i=1}^{N} x_{ti} r_{ti2}, \quad (14)$$

$$\sum_{i=1}^{N} x_i = 1, x_i \geq 0, i = \overline{1,N}, t = \overline{1,T} \quad (15)$$

3. $0 < b < 1$

From (4) it is evident that this case is possible when $\frac{1}{T}\sum_{i=1}^{T}\sum_{i=1}^{N} x_{ti} r_{ti1} \leq r^* < \frac{1}{T}\sum_{i=1}^{T}\sum_{i=1}^{N} x_{ti}\tilde{r}_{ti}$, or when $\frac{1}{T}\sum_{i=1}^{T}\sum_{i=1}^{N} x_{ti}\tilde{r}_{ti} \leq r^* < \frac{1}{T}\sum_{i=1}^{T}\sum_{i=1}^{N} x_{ti} r_{ti2}$.

a) Let $\frac{1}{T}\sum_{i=1}^{T}\sum_{i=1}^{N} x_{ti} r_{ti1} \leq r^* < \frac{1}{T}\sum_{i=1}^{T}\sum_{i=1}^{N} x_{ti}\tilde{r}_{ti}$.

Then using (4) – (7) problem (7) – (9) is reduced to the following problem of nonlinear programming:

$$\tilde{r} = \frac{1}{T}\sum_{i=1}^{T}\sum_{i=1}^{N} x_{ti}\tilde{r}_{ti} \to \max, \quad (16)$$

$$\beta = \frac{1}{\frac{1}{T}\sum_{i=1}^{T}\sum_{i=1}^{N} x_{ti} r_{ti2} - \frac{1}{T}\sum_{i=1}^{T}\sum_{i=1}^{N} x_{ti} r_{ti1}} \left(r^* - \frac{1}{T}\sum_{i=1}^{T}\sum_{i=1}^{N} x_{ti} r_{ti1} \right) +$$

$$+ \frac{1}{\frac{1}{T}\sum_{i=1}^{T}\sum_{i=1}^{N} x_{ti} r_{ti2} - \frac{1}{T}\sum_{i=1}^{T}\sum_{i=1}^{N} x_{ti} r_{ti1}} \cdot$$

$$\cdot \left(\frac{1}{T}\sum_{i=1}^{T}\sum_{i=1}^{N} x_{ti}\tilde{r}_{ti} - r^* \right) \cdot \ln \left(\frac{\frac{1}{T}\sum_{i=1}^{T}\sum_{i=1}^{N} x_{ti}\tilde{r}_{ti} - r^*}{\frac{1}{T}\sum_{i=1}^{T}\sum_{i=1}^{N} x_{ti}\tilde{r}_{ti} - \frac{1}{T}\sum_{i=1}^{T}\sum_{i=1}^{N} x_{ti} r_{ti1}} \right), \quad (17)$$

$$\frac{1}{T}\sum_{i=1}^{T}\sum_{i=1}^{N} x_{ti} r_{ti1} \leq r^*, \quad (18)$$

$$\frac{1}{T}\sum_{i=1}^{T}\sum_{i=1}^{N} x_{ti}\tilde{r}_{ti} > r^*, \quad (19)$$

$$\sum_{i=1}^{N} x_i = 1, x_i \geq 0, i = \overline{1,N}, t = \overline{1,T} \quad (20)$$

b) Let $\frac{1}{T}\sum_{i=1}^{T}\sum_{i=1}^{N} x_{ti}\tilde{r}_{ti} \leq r^* < \frac{1}{T}\sum_{i=1}^{T}\sum_{i=1}^{N} x_{ti}r_{ti2}$.

Then the problem (7) – (9) is reduced to the following problem of nonlinear programming:

$$\tilde{r} = \frac{1}{T}\sum_{i=1}^{T}\sum_{i=1}^{N} x_{ti}\tilde{r}_{ti} \to \max, \quad (21)$$

$$\beta = \frac{1}{\frac{1}{T}\sum_{i=1}^{T}\sum_{i=1}^{N} x_{ti}r_{ti2} - \frac{1}{T}\sum_{i=1}^{T}\sum_{i=1}^{N} x_{ti}r_{ti1}}\left(r^* - \frac{1}{T}\sum_{i=1}^{T}\sum_{i=1}^{N} x_{ti}r_{ti1}\right) -$$

$$- \frac{1}{\frac{1}{T}\sum_{i=1}^{T}\sum_{i=1}^{N} x_{ti}r_{ti2} - \frac{1}{T}\sum_{i=1}^{T}\sum_{i=1}^{N} x_{ti}r_{ti1}} \cdot$$

$$\cdot \left(r^* - \frac{1}{T}\sum_{i=1}^{T}\sum_{i=1}^{N} x_{ti}\tilde{r}_{ti}\right) \cdot \ln\left(\frac{r^* - \frac{1}{T}\sum_{i=1}^{T}\sum_{i=1}^{N} x_{ti}\tilde{r}_{ti}}{\frac{1}{T}\sum_{i=1}^{T}\sum_{i=1}^{N} x_{ti}r_{ti2} - \frac{1}{T}\sum_{i=1}^{T}\sum_{i=1}^{N} x_{ti}r_{ti1}}\right), \quad (22)$$

$$\frac{1}{T}\sum_{i=1}^{T}\sum_{i=1}^{N} x_{ti}r_{ti2} > r^*, \quad (23)$$

$$\frac{1}{T}\sum_{i=1}^{T}\sum_{i=1}^{N} x_{ti}\tilde{r}_{ti} \leq r^*, \quad (24)$$

$$\sum_{i=1}^{N} x_i = 1, x_i \geq 0, i = \overline{1,N}, t = \overline{1,T} \quad (25)$$

The R-algorithm of minimization of not differentiated functions is applied to the decision of problems (16) – (20) and (21) – (25). Vector $x = \{x_i\}$ $i = \overline{1,N}$ is the required portfolio structure.

VI. Experimental Investigations and Result Analysis

As the input data closing prices of leading companies at the stock exchange NYSE, the authors used: Canon Inc. (ADR), McDonald's Corporation (MCD), PepsiCo, Inc (PEP), the Procter & Gamble Company (PG), SAP AG (ADR) (SAP) in the period from 13 December 2013 to 17 January 2014. The founded profitableness is presented in Table I.

TABLE I

PROFITABLENESS, %

Companies / Dates	CAJ	MCD	PEP	PG	SAP
13 December 2012	1.650	1.337	2.128	2.843	1.566
20 December 2012	-0.060	-1.111	-1.000	-0.184	-2.247
27 December 2012	1.560	-0.633	-1.038	-0.861	-1.132
3 December 2014	0.880	0.484	0.808	1.890	2.655
10 January 2014	1.770	0.052	-1.483	0.422	1.042
17 January 2014	**-1.270**	**-0.105**	**0.206**	**0.162**	**0.843**

Further using the Fuzzy GMDH method [10] with triangular membership functions, linear partial descriptions and training sample of 70%, the next profitableness values were forecasted for each week (Table II):

TABLE II

FORECASTED VALUES OF SHARES PROFITABLENESS, %

Companies	Profitableness				MAPE test sample	MSE Test sample
	Low bound	Forecasted	Upper bound	Real		
13 December 2012						
CAJ	1.498	1.636	1.774	1.65	2.5877	0.0163
MCD	1.112	1.351	1.59	1.337	2.8177	0.0325
PEP	2.015	2.156	2.297	2.128	2.7129	0.0233
PG	2.61	2.839	3.068	2.843	2.3729	0.0088

SAP	1.387	1.599	1.811	1.566	1.7877	0.0364
20 December 2013						
CAJ	-0.284	-0.036	0.212	-0.06	1.4456	0.0235
MCD	-1.303	-1.144	-0.985	-1.111	2.1045	0.0302
PEP	-1.265	-0.994	-0.723	-1	1.0498	0.0131
PG	-0.35	-0.171	0.008	-0.184	2.4122	0.0162
SAP	-2.309	-2.207	-2.105	-2.247	2.5935	0.0348
27 December 2013						
CAJ	1.356	1.594	1.832	1.56	2.1562	0.0212
MCD	-0.846	-0.647	-0448	-0.633	1.8341	0.0287
PEP	-1.167	-1.006	-0.845	-1.038	2.6743	0.0189
PG	-1.134	-0.875	-0.616	-0.861	2.0452	0.0209
SAP	-1.305	-1.103	-0.901	-1.132	1.8243	0.0152
3 January 2014						
CAJ	0.696	0.914	1.132	0.88	2.4284	0.0283
MCD	0.218	0.497	0.776	0.484	1.9135	0.0134
PEP	0.604	0.796	0.988	0.808	1.899	0.0092
PG	1.662	1.863	2.064	1.89	2.2715	0.0088
SAP	2.539	2.683	2.827	2.655	1.8143	0.0256
10 January 2014						
CAJ	1.498	1.746	1.994	1.77	1.5463	0.0074
MCD	-0.16	0.027	0.214	0.052	1.5162	0.0147
PEP	-1.657	-1.512	-1.367	-1.483	2.1347	0.0382
PG	0.214	0.447	0.68	0.422	2.1756	0.0301
SAP	0.846	1.024	1.202	1.042	2.0015	0.0215
17 January 2014						
CAJ	-1.484	-1.246	-1.008	-1.27	2.5877	0.0163
MCD	-0.347	-0.118	0.111	-0.105	2.8177	0.0325
PEP	0.001	0.242	0.483	0.206	2.7129	0.0233
PG	0.041	0.17	0.299	0.162	2.3729	0.0088
SAP	0.675	0.867	1.059	0.843	1.7877	0.0364

By using the Fuzzy GMDH method, the portfolio optimization system stops to be dependent on the factor of expert subjectivity. Let us see the results of application of the suggested approach to determining an optimal invest portfolio to the date 17 January 2014. Let the critical profitableness level set by trader 0.7 %. Varying the risk level we obtain the following results for triangular MF presented in Tables III, IV and Fig. 1. As we can see in Fig. 1, the dependence of profitableness–risk has descending type; the greater risk, the lesser profitableness is opposite to classical probabilistic methods. It may be explained by the fact that in the fuzzy approach by risk is meant the situation when the expected profitableness happens to be less than the given criteria level. When the expected profitableness decreases, the risk grows.

TABLE III

COMPONENTS OF OPTIMAL PORTFOLIO FOR TRIANGULAR MF WITH CRITICAL LEVEL R^*=0.7%

CAJ	MCD	PEP	PG	SAP
0.05482	**0.00196**	**0.0027**	**0.00234**	**0.93818**
0.06145	0.00113	0.00606	0.0039	0.92746
0.0698	0.00577	0.00235	0.00219	0.91989
0.06871	0.00228	0.0057	0.00244	0.92087
0.07567	0.00569	0.00106	0.00094	0.91664
0.07553	0.00002	0.0029	0.00208	0.91947
0.06774	0.00121	0.006	0.00234	0.92271
0.0764	0.001	0.00612	0.00464	0.91184
0.09072	0.00849	0.00655	0.0039	0.89034

TABLE IV

PARAMETERS OF OPTIMAL PORTFOLIO WITH CRITICAL LEVEL r^*=0.7%

Low bound	Expected profitableness	Upper bound	Risk level
0.55133	**0.74591**	**0.94049**	**0.2**
0.53462	0.72954	0.92446	0.25
0.51544	0.71084	0.90624	0.3
0.51894	0.71431	0.90968	0.35
0.5045	0.70018	0.89587	0.4
0.50877	0.70425	0.89973	0.45
0.522	0.71731	0.91262	0.5
0.50197	0.69752	0.89308	0.55
0.46358	0.66014	0.8567	0.6

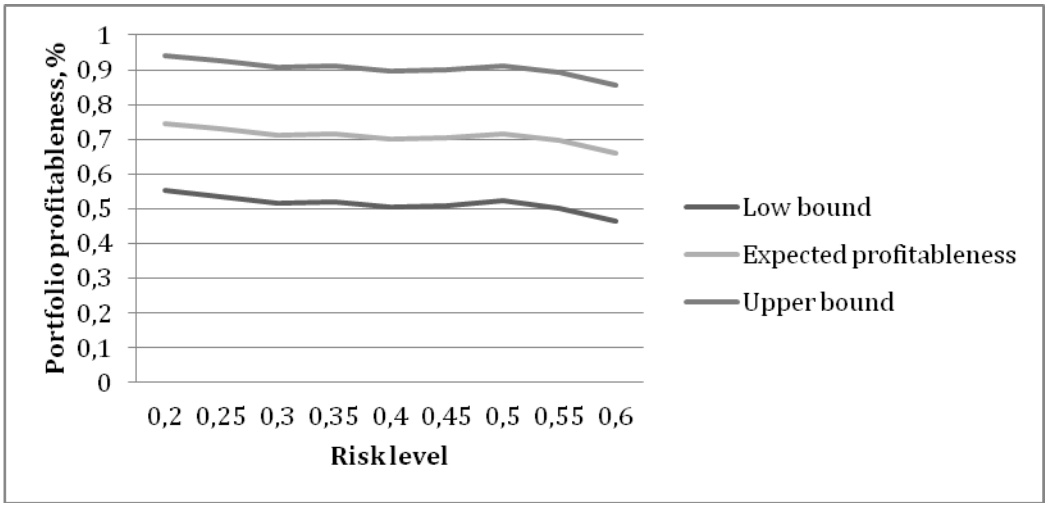

Fig. 1. Dependence of expected portfolio profitableness on risk level for triangular MF.

The calculated corridor of profitableness for optimal portfolio is [0.55133; 0.74591; 0.94049]. The main portfolio portion in this case goes to company SAP that can be explained by the high level of its profitableness in comparison with other companies.

Now consider an optimal portfolio during 6 weeks (see Tables V, VI and Fig. 2).

TABLE V

OPTIMAL PORTFOLIO COMPONENTS DURING TIME PERIOD AND CRITERIA LEVEL 0.7%

CAJ	MCD	PEP	PG	SAP
0.00558	**0.00123**	**0.00072**	**0.94771**	**0.06476**
0.00871	0.00245	0.00051	0.92732	0.06101
0.00629	0.00595	0.00137	0.92718	0.05921
0.00651	0.00332	0.00058	0.91748	0.07211
0.00634	0.00893	0.00075	0.91475	0.06923
0.00877	0.00094	0.00032	0.92444	0.06553
0.00641	0.00056	0.00162	0.91957	0.07184
0.00746	0.00239	0.00054	0.90852	0.08109
0.00901	0.00654	0.00021	0.88581	0.09843

TABLE VI

OPTIMAL PORTFOLIO COMPONENTS DURING TIME PERIOD AND CRITERIA LEVEL 0.7%

Low bound	Expected profitableness	Upper bound	Risk level
0.49289	**0.69582**	**0.89876**	**0.2**
0.47619	0.68156	0.88693	0.25
0.47517	0.68102	0.88687	0.3
0.45507	0.66089	0.86671	0.35
0.46821	0.67435	0.88049	0.4
0.45918	0.66511	0.87104	0.45
0.45098	0.65674	0.8625	0.5
0.43601	0.64202	0.84803	0.55
0.42386	0.63087	0.83788	0.6

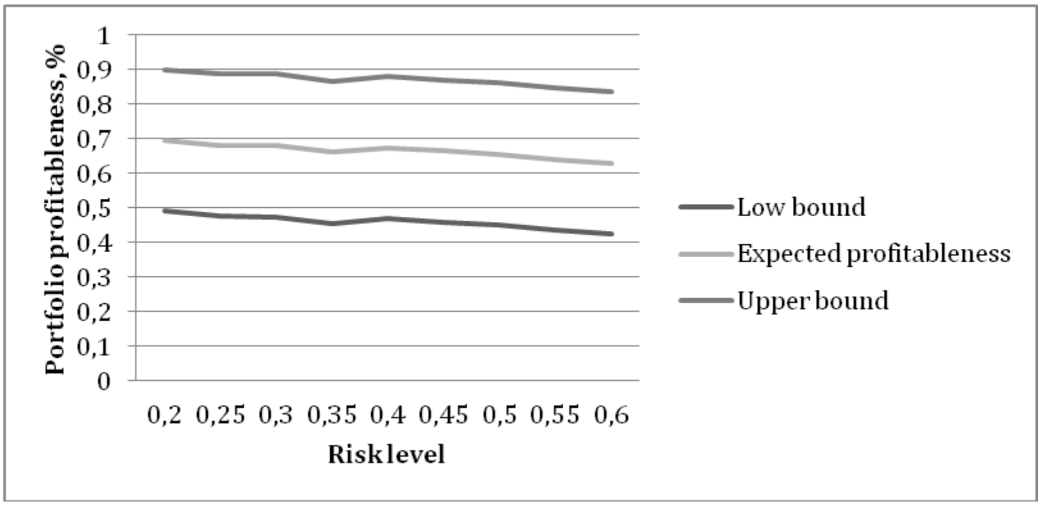

Fig. 2. The dependence of expected portfolio profitableness on risk level.

The calculated corridor of profitableness for optimal portfolio is [0.49289; 0.69582; 0.94049].

The main portfolio portion in this case goes to company PG that can be explained by the high level of its average profitableness in comparison with other companies.

VII. CONCLUSION

In this paper, the research in the field of portfolio management has been carried out. Fuzzy set theory has been used as a tool for getting an optimal portfolio. s a result of research, the mathematical model based on the fuzzy-set approach for finding of structure of the optimal investment portfolio has been received. On the basis of the theory of fuzzy sets, the algorithm of optimization of a share portfolio has been developed. As a result of research, the following conclusions have been made:

1. The dependence of profitableness–risk has a descending type; the greater risk, the lesser profitableness is opposite to classical probabilistic methods.
2. Portfolios during the time period and at the end of period have different structure and characteristics that can be explained by the different calculations of profitableness.
3. For improving the accuracy of the suggested fuzzy portfolio model, the fuzzy GMDH method has been applied to profitableness forecasting. The experimental investigations have proved its high efficiency.

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Inna A. Sydoruk graduated from the Institute for Applied System Analysis, NTUU "Kiev Polytechnic Institute". Her major fields of research are: information systems, fuzzy set theory, investment portfolio optimization, decision-making theory.
E-mail: isidoruk@ukr.net.

Yuriy P. Zaychenko is an Associate Professor with Kiev Polytechnic Institute. Since 1981, he has been a Professor the Institute for Applied System Analysis of Kiev Polytechnic Institute. His main fields of research are: decision- making under uncertainty and risk, fuzzy logic systems and fuzzy neural network and their applications in economy and finance, computational intelligence. Since 1995, he has been an Academician of the Ukrainian Academy of High School. He received a state prize award in science and technology in 2012.
E-mail: zaychenkoyuri@ukr.net, baskervil@voliacable.com.

Inna Sidoruka, Jurijs Zaičenko. Investīciju portfeļa optimizācija laika periodā
Investīciju portfeļa izmantošanas galvenais uzdevums ir uzlabot investēšanas nosacījumus, piešķirot vērtspapīru (VP) kopumam tādus investīciju raksturlielumus, kuri nav sasniedzami, izmantojot tikai atsevišķu VP, bet ir iespējami tikai gadījumā, kad tos izmanto kombinācijā. Mūsdienu pasaules ekonomisko procesu nestabilitāte rada neskaidrību investīciju lēmumu pieņemšanā. Sakarā ar nepieciešamību cīnīties pret to, šajā darbā tiek izmantota izplūdušo kopu pieeja, kur portfeļa risks ir nevis tā nestabilitāte, bet gan iespējamība, ka sagaidāmais portfeļa ienesīgums izrādīsies zemāks par kādu iepriekš definētu plānoto vērtību. Portfeļa aktīvu korelācija netiek apskatīta un ņemta vērā. Katra aktīva ienesīgums nav gadījumveida izplūdusī vērtība (piemēram, trīsstūra vai intervāla veidā). Šajā darbā apskatīts investīciju portfeļa optimizācijas uzdevums konkrētā periodā. Ir jānosaka portfeļa struktūra, kura sniegs maksimālo vidējā ienesīguma līmeni periodā, ņemot vērā noteikto riska līmeni. Izejas dati tika iegūti no NYSE biržas, ņemot tādu ietekmīgu uzņēmumu kā Canon Inc. (ADR), McDonald's Corporation (MCD), PepsiCo, Inc. (PEP), Procter & Gamble Company (PG), SAP AG (SAP) akciju cenas izsolē pirms slēgšanas, laika posmā no 13.12.13. līdz 17.01.14. Ienesīguma prognozēšanai tika izmantota argumentu grupveida uzskaites izplūdusī metode, kas ļāva izvairīties no ekspertu subjektivitātes sistēmā. Rezultātā tika izveidots optimālais portfelis sestās nedēļas beigām (17.01.14.) un visam laika posmam. Veikta iegūto rezultātu salīdzinošā analīze. Šādā veidā darbā tiek turpināts pētījums par investēšanas vadību, izmantojot kvalitatīvi jaunu izplūdušo kopu pieeju. Balstoties uz izveidoto matemātisko modeli un, izmantojot tā realizāciju programmatūras veidā, tika veikta noteiktajam laika periodam optimālā portfeļa meklēšana un iegūto rezultātu analīze.

Инна Сидорук, Юрий Зайченко. Оптимизация инвестиционного портфеля в течение периода времени
Основная задача портфельного инвестирования –улучшить условия инвестирования, предоставляя совокупности ценных бумаг (ЦБ) такие инвестиционные характеристики, которые недостижимы с позиции отдельно взятой ЦБ и возможны только при их комбинации. Нестабильность экономических процессов современного мира порождает неопределенность в процессе принятия инвестиционных решений. В связи с потребностью в борьбе с неопределенностью в работе применяется нечетко-множественный подход, где риск портфеля – это не его волатильность, а возможность того, что ожидаемая доходность портфеля окажется ниже некоторой установленной плановой величины; корреляция активов в портфеле не рассматривается и не учитывается; доходность каждого актива – это неслучайное нечеткое число (например, треугольного или интервального вида). В текущей работе рассмотрена прямая задача оптимизации инвестиционного портфеля для заданного периода. Нужно определить структуру портфеля, который обеспечит максимальный уровень средней доходности за период при заданном уровне риска. В качестве исходных данных были использованы цены закрытия на бирже NYSE таких влиятельных компаний, как Canon Inc. (ADR), McDonald's Corporation (MCD), PepsiCo, Inc (PEP), the Procter & Gamble Company (PG), SAP AG (ADR) (SAP) в период с 13.12.13 по 17.01.14. Для прогнозирования доходности был использован нечеткий метод группового учета аргументов (НМГУА), что позволило лишить систему субъективности экспертов. В результате был построен оптимальный портфель на конец шестой недели (17.01.14) и для всего периода времени. Проведен сравнительный анализ полученных результатов. Таким образом, в работе продолжено исследование в области управления инвестиционной деятельностью с использованием качественно нового нечетко-множественного подхода. На основе построенной математической модели был программно реализован поиск оптимального портфеля для заданного периода и проанализированы полученные результаты.