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The Comparison of Approaches Used for Estimating Uncertain Probabilities

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Abstract **– Probabilistic estimates are numerical representations of chances of random event occurrence. The classical theory of probability is based on the assumption that probabilistic estimates are deterministic. If available initial data are sufficient, this kind of estimates can be really obtained. However, when such data are not available, probabilistic estimates become uncertain. This paper analyses and compares three widespread approaches to modelling uncertain estimates and provides practical recommendations on their use.**

Keywords **– Choquet capacities, interval probabilities, lower and upper probabilities, λ-measures, monotone measures, Möbius representation**

I. Introduction

The classical theory of probability postulates that the probabilities of random events have to be determined unambiguously. This postulate underlies all other operations on probabilities, e.g., calculation of probabilities for unions and intersections of the sets of random events, recalculation of the posterior probabilities according to Bayes' theorem, probabilistic inference on the networks etc. It is true that probabilities of relevant events can be determined unambiguously if sufficient initial information is available. However, in real life, suitable conditions for obtaining deterministic probabilistic estimates are not always ensured. Plenty of examples can be provided when objective probabilistic estimates are not confident. One evident example could be estimating the safety of technical system operation. Quite frequently, the probabilities of technical system component failures are evaluated on the basis of insufficient statistical information. Besides, not always all the factors are taken into account that might somehow affect the functioning of the estimated component. The use of deterministic probabilistic estimates in cases like that is just an attempt to shut your eyes to the problem. The matter with the assessment of ecological risks looks even worse. The diversity of components of ecosystems and the complexity and insufficient knowledge of the complicated links between the components can lead to a situation when probability estimate of the harm that might be made to a certain component as a result of the technogenic disaster might not represent real state of things at all.

A lot of examples of other kinds can be given as well. For example, the probability estimates of profit level earned through investing capital into securities, made on the basis of the available information, may become meaningless due to various fluctuations and upheavals in the financial market. Estimates of probabilities of the development of the political situation in a region might also be unreliable due to the effect of multiple unknown factors, but the estimates of the chances of candidates for the position of President to win might change essentially even due to reckless statements of one of candidates.

Even if due to some reasons probabilistic estimates cannot be assigned unambiguously, two boundary estimates are assigned for each element of the relevant set. Sets of such estimates constitute two boundary probabilistic distributions. A problem then arises how to manage these uncertain probabilistic distributions. This paper examines some approaches most frequently used to solve that task.

II. Fundamentals of the Theory of Probability Evaluation

Quite frequently uncertainty is an inherent attribute of information. Different types of uncertainty exist; to correctly cope with uncertain information, it is necessary to measure uncertainties correctly. Although the notion of uncertainty is quite specific, it has to be measured according to general regulations and requirements of general theory of measurement.

Nowadays the general theory of measurement represents a developed scientific and applied discipline, whose main goals are correct measurements of attributes and properties of different objects, processes and occurrences; determination of suitable measurement scales and permissible transformations of the numbers expressing results of particular measurements. A detailed and competent presentation of the fundamentals of measurement theory can be found in [4], while more specific issues related to measuring probabilities are discussed in [1].

Let us first consider classical additive estimates. Let a universal set X be specified, in which a non-empty set (family) of subsets A is defined. It is assumed that these subsets have a suitable algebraic structure. For example, in the case of classical probabilistic estimates it is assumed that the structure of subsets in X possesses properties of σ-algebra. Under these conditions, ***classical estimate*** is an estimate

$$\nu : A \to [0, \infty],$$

which possesses these properties:

1. $\nu(\varnothing) = 0$;
2. For each sequence B_1, B_2 … of pairwise not-connected subsets $B_i \subset A$;

$$\text{if } \bigcup_{i=1}^{\infty} B_i \subset A, \text{ then } \nu\left(\bigcup B_i\right) = \sum_{i=1}^{\infty} \nu\left(B_i\right). \quad (1)$$

Property 2 is a distinguishing feature of all classical estimates. It is called ***countable additivity***. The property of additivity is a strong requirement and in many cases it is not met. To enlarge possibilities of measuring uncertainties, other more general estimates are needed. It can be achieved by replacing the restrictive requirement of additivity for classical estimates with a weaker requirement of monotonicity.

Let us define a class of monotone estimates in a family of sets A of the universal set X as follows:

$$\nu : A \Rightarrow [0, \infty].$$

The monotone estimates have to possess these properties:

1. $\nu(\varnothing) = 0.$

2. For any $B_i, B_j \subset A$, if $B_i \subseteq B_j$, $\nu(B_i) \le \nu(B_j)$.

For any monotone estimate, if $B_i \subseteq A, B_j \subseteq A, B_i \bigcup B_j \subseteq A$,

$$\nu\left(B_i \bigcap B_j\right) \le \min\left\{\nu\left(B_i\right), \nu\left(B_j\right)\right\}. \quad (2)$$

$$\nu\left(B_i \bigcup B_j\right) \ge \max\left\{\nu\left(B_i\right), \nu\left(B_j\right)\right\}. \quad (3)$$

If, besides inequalities (2) and (3) for disconnected subsets $B_i, B_j \subseteq A$ and their union $B_i \bigcup B_j \subseteq A$ the following inequality holds:

$$\nu\left(B_i \bigcup B_j\right) \ge \nu\left(B_i\right) + \nu\left(B_j\right), \quad (4)$$

the estimate ν is called ***superadditive***.
If under the same conditions this inequality holds:

$$\nu\left(B_i \bigcup B_j\right) \le \nu\left(B_i\right) + \nu\left(B_j\right), \quad (5)$$

the estimate ν is called ***subadditive***.

Depending on different purposes, sometimes a necessity to transform original monotonous estimates to another form appears. Such a necessity can be caused for example by studying properties of new kind of estimates. One of widely known transformations of that kind is ***Möbius transformation***. Let us assume that a finite set of elements *X* is specified and denote a set of all possible subsets of *X* as $\mathbf{F}(X)$. Let ν be a set function defined in subsets *X* that stands for the estimate function of the measured attribute. In its standard form, the process of measurement is reduced to the representation of intensity (strength) of the measured attribute in $\mathbf{F}(X)$ to a set of real numbers:

$$\nu : \mathbf{F}(X) \to \mathbf{R}.$$

The set function ν can be correctly transformed into another set function

$$m^{\nu} : \mathbf{F}(X) \to \mathbf{R}.$$

by means of Möbius transformation

$$m^{\nu}(B) = \sum_{B \subseteq A} (-1)^{|A-B|} \nu(B). \quad (6)$$

Function m^{ν} is called ***Möbius representation*** for ν (alternatively, m^{ν} is called ***Möbius function***).

If ***Möbius representation***, m^{ν}, is known, the initial function ν can be uniquely determined by means of inverse transformation

$$\nu(B) = \sum_{A \subseteq B} m^{\nu}(A). \quad (7)$$

Let us examine some important relationships between the set function ν and its Möbius representation m^{ν}. The set function ν represents monotone estimates, if its Möbius representation possesses these properties:

1. $m^{\nu}(\varnothing) = 0.$
2. $\sum_{B \subseteq \mathbf{F}(X)} m^{\nu}(B) = 1.$
3. $\sum_{A \subseteq B} m^{\nu}(A) \ge 0$ for all $B \subseteq \mathbf{F}(X)$.

The concept of monotone estimates is quite a broad notion. A special class of monotonous estimates is formed of estimates called Choquet capacities. Their essence is as follows. Let us assign an integer $k \ge 2$. The Choquet capacity of the k-th order is a monotone estimate ν that satisfies these inequalities:

$$\nu\left(\bigcup_{i=1}^{k} B_i\right) \ge \sum_{\substack{k \subseteq N_k \\ k \neq \varnothing}} (-1)^{|k|+1} \nu\left(\bigcap_{i \in k} B_i\right) \quad (8)$$

for all families of k subsets X. By agreement, any monotone estimate that does not satisfy inequalities (8) is formally ascribed to Choquet capacities of the 1st order.

To subsets B_i in expression (8) a requirement of non-connection is posed, i.e., they must not overlap. In the general case, at the overlapping subsets expression (8) looks differently. For example, for $k = 2$ we have

$$\nu\left(B_1 \bigcup B_2\right) \ge \nu\left(B_1\right) + \nu\left(B_2\right) - \nu\left(B_1 \bigcap B_2\right). \quad (9)$$

In the case of the overlapping subsets, the most general character has Choquet capacities of arbitrary order $k \ge 2$

$$\nu\left(B_1 \bigcup B_2 \bigcup \ldots \bigcup B_n\right) \ge \sum_i \nu\left(B_i\right) - \sum_{i<j} \begin{matrix} \nu\left(B_i \bigcap B_j\right) + \ldots \\ + (-1)^{k+1} \nu\left(B_1 \bigcap B_2 \bigcap \ldots \bigcap B_k\right) \end{matrix} . \quad (10)$$

There are important connections between Möbius representations and Choquet capacities of order $k \ge 2$. Let us consider the most essential connections of that kind

1. If ν is Choquet capacity of order $2 \le |B| \le k$, then $m^{\nu}(B) \ge 0$.
2. ν is Choquet capacity of order ∞, if $m^{\nu}(B) \ge 0$ for all $B \in \mathbf{F}(X)$.
3. ν is Choquet capacity of order $k \ge 2$, if $\sum_{C \subseteq A \subseteq D} m^{\nu}(A) \ge 0$ for all $B \in \mathbf{F}(X)$ and all $C \in \mathbf{F}(X)$, such that $2 \le C \le k$.

III. Conceptual Principles of Uncertain Probabilities

Let us introduce the following denotations:

X – the finite set (universe of discourse) of elementary random events;

$\mathbf{F}(X)$ – the set of all subsets X ;

φ – the set of probability distribution functions in X . (In literature, this set is frequently called a ***credal set***).

Using the above system of denotations, ***the lower probability function*** for all sets $A \in \mathbf{F}(X)$ is determined as follows:

$$\underline{p}^{\varphi}(A) = \inf_{p\in\varphi} \sum_{x\in A} p(x). \tag{11}$$

By analogy, ***the upper probability function*** is determined as follows:

$$\overline{p}^{\varphi}(A) = \sup_{p\in\varphi} \sum_{x\in A} p(x). \tag{12}$$

Let us give without proof a summary of properties of lower and upper probabilities.

$$\underline{p}^{\varphi} \geq \overline{p}^{\varphi} \text{ for all } A \in \mathbf{F}(X). \tag{13}$$

$$\underline{p}^{\varphi}(\varnothing) = \overline{p}^{\varphi} = 0. \tag{14}$$

$$\underline{p}^{\varphi}(X) = \overline{p}^{\varphi}(X) = 1. \tag{15}$$

$$\overline{p}^{\varphi}(A) = 1 - \underline{p}^{\varphi}(A) \text{ for all } A \in \mathbf{F}(X). \tag{16}$$

$$\underline{p}^{\varphi}(A \cup B) \geq \underline{p}^{\varphi}(A) + \underline{p}^{\varphi}(B) \text{ for all } A, B \in \mathbf{F}(X), A \cap B = \varnothing. \tag{17}$$

Expression (17) shows that the lower probability functions are superadditive

$$\overline{p}^{\varphi}(A \cup B) \leq \overline{p}^{\varphi}(A) + \overline{p}^{\varphi}(B) \tag{18}$$

for all $A, B \in \mathbf{F}(X), A \cap B = \varnothing$.

From expression (18) it follows that the upper probability functions are subadditive.

The lower and upper probabilities can be subjected to Möbius transformation (see Expression (6)). As a result, two Möbius representations (functions), $\underline{m}$ and $\overline{m}$ are obtained. Since functions $\underline{p}$ and $\overline{p}$ are dual, functions $\underline{m}$ and $\overline{m}$ are also dual. The correlation between those dual representations for any set $A \in \mathbf{F}(X)$ is expressed as follows:

$$\overline{m}(A) = (-1)^{|A|+1} \sum_{B \subseteq A} \underline{m}(B). \tag{19}$$

The lower and upper conditional probabilities are determined as follows:

$$\underline{p}(A/B) = \inf_{p\in\varphi} \frac{\sum_{x\in A\cap B} p(x)}{\sum_{x\in B} p(x)} \text{ for all } A, B \in \mathbf{F}(X); \tag{20}$$

$$\overline{p}(A/B) = \sup_{p\in\varphi} \frac{\sum_{x\in A\cap B} p(x)}{\sum_{x\in B} p(x)} \text{ for all } A, B \in \mathbf{F}(X). \tag{21}$$

Let us consider the main correlations between marginal and joint uncertain probabilities. Let us have the Cartesian product of two probabilistic spaces $X \times Y$. Let us denote the lower joint probabilistic function in $X \times Y$ as $\underline{p}$, the upper probabilistic function in $X \times Y$ as $\overline{p}$ but a set of probability distributions compatible with $\underline{p}$ and $\overline{p}$ as φ. Let us denote lower marginal probabilities in X and Y as $\underline{p}_X$, $\underline{p}_Y$; upper marginal probabilities in X and Y as $\overline{p}_X$, $\overline{p}_Y$ and sets of marginal probability distributions compatible with $\underline{p}_X$, $\overline{p}_X$ and $\underline{p}_Y$, $\overline{p}_Y$, respectively, as φ_X , φ_Y Sets φ_X , φ_Y can be determined using these expressions:

$$\varphi_x = \left\{ p_X / p_X(x) = \sum_{y\in Y} p(x,y) \right\} \text{ for some distribution } p \in \varphi; \tag{22}$$

$$\varphi_Y = \left\{ p_Y / p_Y(y) = \sum_{x\in X} p(x,y) \right\} \text{ for some distribution } p \in \varphi. \tag{23}$$

The marginal lower probabilities are defined as follows:

$$\underline{p}_X(A) = \underline{p}(A \times Y) \text{ for all } A \in \mathbf{F}(X); \tag{24}$$

$$\underline{p}_Y(B) = \underline{p}(X \times B) \text{ for all } B \in \mathbf{F}(Y). \tag{25}$$

The marginal upper probabilities are defined as follows:

$$\overline{p}_X(A) = \overline{p}(A \times Y) \text{ for all } A \in \mathbf{F}(X); \tag{26}$$

$$\overline{p}_Y(B) = \underline{p}(X \times B) \text{ for all } B \in \mathbf{F}(Y). \tag{27}$$

The marginal Möbius functions are determined as follows:

$$m_X(A) = \sum_{S / A \in S_X} m(S_X) \text{ for all } A \in \mathbf{F}(X); \tag{28}$$

$$m_Y(B) = \sum_{S / B \in S_Y} m(S_Y) \text{ for all } B \in \mathbf{F}(X), \tag{29}$$

where $S_X = \{x \in X / (x,y) \in S \text{ for some } y \in Y\}$;

$S_Y = \{y \in Y / (x,y) \in S \text{ for some } x \in X\}$.

From the above analysis it directly follows that all operations that are valid for classical probabilities can also be performed on uncertain probabilities. It is clear that taking into account the specifics of uncertain probabilities, operations on them can be quite complicated.

IV. INTERVAL PROBABILITIES

Interval probabilities are a special case of the lower and upper probabilities considered in the previous section. The theory of interval probabilities was first described in [2].

Let us introduce some definitions. Let there be a set of elements *X* (random events, values of random variable), $X=\{x_1,...,x_n\}$. To each element $x_i \in X$ there is connected an interval of possible probability values, $[l_i,u_i]$, where l_i is the lower boundary but u_i is the upper boundary of that interval. Let us denote a set (family) of such intervals for all elements $x_i \in X$ as $L=\{[l_i,u_i]/i=1,...,n\}$. The values l_i and u_i have to meet the requirement: $0 \le l_i \le u_i \le 1, \forall i=1,...,n$. Sets of interval boundary values can be interpreted as the lower and the upper probability distributions in a set of potentially possible probability distributions

$$\mathcal{P}=\{P \in \mathcal{P}(X)/l_i \le p(x_i) \le u_i, \forall i=1,..,n\}. \quad (30)$$

In other words, L is a set of probability intervals but $\mathcal{P}$ is a set of possible probability distributions correlated to L. This statement forms a direct connection with the concepts of interval probabilities and the general concept of lower and upper probabilities.

To avoid situations when a set of potential probability distributions is empty, $\mathcal{P}=\varnothing$, the following limitations are posed on the values of probability interval boundaries

$$\sum_{i=1}^{n} l_i \le 1 \le \sum_{i=1}^{n} u_i. \quad (31)$$

Probability intervals that satisfy limitations (30) are called ***proper*** intervals.

For the set of potential possible probability distributions, $\mathcal{P}$, defined by means of proper intervals for some set $A \subseteq X$ the lower and the upper functions can be determined using these expressions:

$$l(A)=\inf_{P\in\mathcal{P}} P(A); \quad (32)$$

$$u(A)=\sup_{P\in\mathcal{P}} P(A). \quad (33)$$

To avoid the compatibility between interval boundaries and the set of potential possible probability distributions, the following conditions have to be met:

$$\sum_{i\neq j} l_j + u_i \le 1, \forall i=1,...,n; \quad (34)$$

$$\sum_{i\neq j} u_j + l_i \le 1, \forall i=1,...,n. \quad (35)$$

Sets of probability intervals satisfying inequalities (34) and (35) are called ***reachable***.

Probability intervals considered in this section belong to a special class of lower and upper probabilities: they are Choquet capacities of order $k=2$, i.e., the following holds:

$$l(A\cup B)+l(A\cap B)\ge l(A)+l(B), \forall A \subseteq X;$$

$$u(A\cup B)+u(A\cap B)\ge u(A)+u(B), \forall A \subseteq X.$$

The last of the above relationships represents the so-called duplicated Choquet capacity of order $k=2$.

Let us consider the procedures of calculating marginal probability intervals out of the assigned intervals of joint probabilities. Let there be given sets of elements (random events, values of a random variable) $X=\{x_1,...,x_n\}$ and $Y=\{y_1,...,y_m\}$. Let us denote a set of probability intervals in $\mathcal{P}=X\times Y$ as $L=\{[l_{ij},u_{ij}]/i=1,...,n, j=1,...,m\}$. In the general case, the lower and the upper marginal distributions can be represented as follows:

$$l_X(A)=l(A\times Y), u_X(A)=u(A\times Y); \quad (36)$$

$$l_Y(B)=l(X\times B), u_Y(B)=u(X\times B). \quad (37)$$

The marginal estimates of the probabilities that are obtained using expressions (36) and (37) fully correspond to the probability estimates that might be obtained via the marginalization of the convex set of probability distributions connected with L. The boundaries of marginal probability intervals $L_X=\{[l_i^X,u_i^X]/i=1,...,n\}$ and $L_Y=\{[l_j^Y,u_j^Y]/j=1,...,m\}$ can be calculated at the known set ***L*** as follows:

$$l_i^X=\max\left\{\sum_{j=1}^{m} l_{ij}, 1-\sum_{k\neq i}\sum_{j=1}^{m} u_{kj}\right\}, i=1,...,n; \quad (38)$$

$$u_i^X=\min\left\{\sum_{j=1}^{m} u_{ij}, 1-\sum_{k\neq i}\sum_{j=1}^{m} l_{kj}\right\}, i=1,...,n; \quad (39)$$

$$l_j^Y=\max\left\{\sum_{i=1}^{n} l_{ij}, 1-\sum_{k\neq i}\sum_{i=1}^{n} u_{ik}\right\}, j=1,...,m; \quad (40)$$

$$u_j^Y=\min\left\{\sum_{i=1}^{n} u_{ij}, 1-\sum_{k\neq i}\sum_{i=1}^{n} l_{ik}\right\}, j=1,...,m. \quad (41)$$

V. λ-MEASURES

λ-measures, or ***λ-measures of Sugeno*** have been proposed by M. Sugeno (see, for example, [5]). A detailed description of these estimates and their properties is provided in [6], while a short description in the context of uncertain probabilities can be found in [3]. To represent uncertain probabilities, monotone estimates p^λ are determined using the following normative requirement. Let X be a universal set of elements (random events, values of a random variable). Let us denote a set of all subsets X as φ. If $A,B\in\varphi$ and $A\cap B=\varnothing$, then

$$p^\lambda(A\cup B)=p^\lambda(A)+p^\lambda(B)+\lambda p^\lambda(A)p^\gamma(B), \quad (42)$$

where $-1<\lambda<\infty$ – the specific parameter.

When X is a finite set and values $p^{\lambda}(\{x_i\})$ are assigned to all singletons $x_i \in X$, for any set $A \in \varphi$ the value $p^{\lambda}(A)$ can be calculated as follows:

$$p^{\lambda}(A) = \frac{1}{\lambda}\left[\prod_{x_i \in X}\left(1+\lambda p^{\lambda}(\{x_i\})\right)-1\right]. \quad (43)$$

Relevant value λ can be determined based on the requirement that $p^{\lambda}(X)=1$. Expression for calculating λ is as follows:

$$1+\lambda = \prod_{x_i \in X}\left[1+\lambda p^{\lambda}(\{x_i\})\right]. \quad (44)$$

Let $p^{\lambda}(\{x_i\})>0$ at least for two elements from X and $p^{\lambda}(\{x_i\})<1$ for all elements $x_i \in X$. Then solving equation (44) produces these results:

- if $\sum_{x_i \in X} p^{\lambda}(\{x_1\})<1$, then λ is a unique solution of the equation; it is located in the interval $(0,\infty)$;
- if $\sum_{x_i \in X} p^{\lambda}(\{x_1\})=1$, then $\lambda=0$ is a single solution of the equation;
- if $\sum_{x_i \in X} p^{\lambda}(\{x_1\})>1$, then λ is a unique solution of the equation that is located in the interval $(-1,0)$.

Any λ-measure can be fully determined by values $p^{\lambda}(\{x_i\})$ on singletons $x_i \in X$. When values $p^{\lambda}(\{x_i\})$ are set for all $x_i \in X$, the value of λ can be calculated by expression (44); then the values $p^{\lambda}(A)$ for all $A \in \varphi$ can be calculated using expression (43). Calculation results can be classified as follows:

- $\sum_{x_i \in X} p^{\lambda}(\{x_1\})=1,\ \lambda=0$, we deal with classical probability estimates;
- $\sum_{x_i \in X} p^{\lambda}(\{x_1\})<1,\ \lambda>0$, the values $p^{\lambda}(\{x_i\})$ are interpreted as lower probabilities;
- $\sum_{x_i \in X} p^{\lambda}(\{x_1\})>1,\ \lambda<0$, the values $p^{\lambda}(\{x_i\})$ are interpreted as upper probabilities.

Hence, the latter expressions establish a direct connection between λ-measures and lower/upper probabilities.

Let p^{λ} - be joint λ-measures, set in subsets $X \times Y$. Then marginal λ-measures can be defined in a standard way:

$$p_X^{\lambda}(A) = p_X^{\lambda}(A \times Y); \quad (45)$$

$$p_Y^{\lambda}(A) = p_Y^{\lambda}(A \times B). \quad (46)$$

For calculating marginal λ-measures at the known joint λ-measures, these expressions are employed:

$$p_X^{\lambda}(\{x\}) = \frac{1}{\lambda}\left[\prod_{y \in Y}\left(1+\lambda p^{\lambda}(\{x,y\})\right)-1\right]; \quad (47)$$

$$p_Y^{\lambda}(\{y\}) = \frac{1}{\lambda}\left[\prod_{x \in X}\left(1+\lambda p^{\lambda}(\{x,y\})\right)-1\right]. \quad (48)$$

VI. A Comparative Analysis of the Approaches

An approach to modelling uncertain probabilities on the basis of arbitrary lower and upper probabilities is the most common one; it enables managing arbitrary lower and upper probabilities. The possibility of representing lower and upper probabilities by means of Möbius transformation allows us to correctly analyse them. Provided that joint lower and upper probability distributions are available, one can calculate lower and upper marginal distributions corresponding to them. If necessary, a reverse task of calculating lower and upper joint probability distributions can be solved using the corresponding marginal distributions.

Interval probabilities are a special case of common lower and upper probabilities. These probabilities are Choquet capacities of order *k=2*. The calculation of interval probabilities is simpler as compared to general calculations of uncertain probabilities. However, if sets of interval probability estimates are not ***reachable***, they have to be transformed to the required form. Unfortunately, from the computational point of view this task is quite complicated. However, when the reachable probability estimates are available, further calculations become less complicated, for example, transformation of joint probability distributions into marginal ones and vice versa.

λ-measures represent a specific kind of uncertain probability estimates. The advantage of λ-measures as applied to the representation of lower and upper probabilities is their simplicity and clear interpretability, but their shortcoming is the necessity to take into account the complexity of relevant calculations for the sets including many elements.

For practical calculations, common uncertain probabilities can be employed in all cases. If the initial probability estimates are reachable interval estimates, the preference should be given to the approach based on interval estimates; whereas in the case when these estimates require additional transformation to the form of reachable interval estimates, the choice between a common approach based on using lower/upper probabilities and an interval probability based approach can be made on the basis of initial data analysis. As regards λ-measures, they have an undoubtful advantage at small amounts of initial data.

All the techniques examined in this paper are only valid if uncertain probabilities are assigned in one and the same set of relevant elements. Otherwise, other approaches to modelling uncertain probabilities have to be applied, e.g., the probability box (p-box) technique.

Since all the above-mentioned uncertain probability measures are in essence monotone, their development has become possible due to the successful development of general theory of monotone uncertainty measures.

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Oļegs Užga-Rebrovs, Gaļina Kuļešova. Nenoteiktu varbūtību novērtēšanas pieeju salīdzinājums
Klasiskās varbūtību teorijas pamatā ir pieņēmums par varbūtējo novērtējumu determinēto raksturu. Taču šādi novērtējumi var tikt iegūti, kad ir pietiekami sākotnējie dati. Praktiskās situācijās bieži ir nepieciešams novērtēt relevanto notikumu varbūtības gan pilnīgas objektīvās informācijas neesamības apstākļos, gan arī tās būtiskas nepietiekamības apstākļos. Dažādu iemeslu dēļ ekspertiem var būt apgrūtinoši noteikt pieprasītos viennozīmīgos varbūtējos novērtējumus. Un pat, ja tādi novērtējumi ir noteikti, novērtējumu ticamība var būt ļoti zema. Liekas priekšroka tiek dota eksplicētā veidā modelēt varbūtējo novērtējumu nenoteiktību. Pēdējos gados ir izstrādātas lietderīgas pieejas nenoteiktu varbūtējo novērtējumu modelēšanai. Šādu pieeju attīstība kļuvusi par iespējamu, pateicoties monotono nenoteiktību novērtējumu teorijas attīstībai. Mēbiusa transformāciju un Šoke kapacitāšu izmantošana deva iespēju uz stingra pamata analizēt dažādu nenoteiktības novērtējumu īpašības. Šajā rakstā izskatītas trīs izplatītas pieejas nenoteiktu varbūtējo novērtējumu modelēšanai: apakšējās un augšējās vispārīgā veida varbūtības, intervālu varbūtības un λ-novērtējumi. Visi šie novērtējumu veidi ir monotonie varbūtējie novērtējumi. Rakstā konspektīvā formā tiek doti apskatīto novērtējumu veidu teorētiskie pamati un tiek analizētas novērtējumu īpašības. Uz analīzes pamata tiek piedāvātas praktiskās rekomendācijas novērtējumu izmantošanai dažādās situācijās.

Олег Ужга-Ребров, Галина Кулешова. Сравнение подходов, используемых для оценивания неопределенных вероятностей
Классическая теория вероятностей базируется на предположении детерминированности вероятностных оценок. Однако, такие оценки могут быть получены при наличии достаточных исходных данных. В практических ситуациях часто необходимо производить оценивание вероятностей релевантных событий либо при полном отсутствии объективной информации, либо при её существенной недостаточности. В таких случаях широко используется экспертное оценивание. По ряду причин экспертам может быть затруднительным назначить требуемые однозначные вероятностные оценки. И даже если такие оценки назначены, их доверительность может быть очень низкой. Представляется предпочтительным эксплицитно моделировать существующую неопределённость относительно вероятностных оценок. В последние годы разработаны действенные подходы к моделированию неопределённых вероятностных оценок. Развитие таких подходов стало возможным в связи с бурным развитием теории монотонных оценок неопределённостей. Использование трансформаций Мёбиуса и мощностей Шоке позволило на строгой основе анализировать свойства различных оценок неопределённостей. В настоящей статье рассматриваются три распространённых подхода к моделированию неопределённых вероятностных оценок: нижние и верхние вероятности общего вида, интервальные вероятности и λ-оценки. Все эти типы оценок являются монотонными вероятностными оценками. В статье в конспективной форме даются теоретические основы рассматриваемых типов оценок и анализируются их свойства. На основе анализа предлагаются практические рекомендации по использованию этих оценок в различных ситуациях.