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Stock Market Structural Changes Discovering Helical Structure of Volatility Wave Fourier Image

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Abstract **– This paper explores an alternative volatility estimation approach discovering the helical structure of Fourier coefficients of volatility wave. Volatility wave is calculated by using wavelet decomposition with consequent logarithmic variance indicator estimation for each decomposed part of the signal and subsequent volatility matrix transform in a specified way. Further, using discrete Fourier transform the Fourier image of obtained volatility wave is analyzed. The Fourier image coefficients of the transformed volatility indicator have a clear helical (spiral) structure that evolves in time. This brings a new understanding of volatility and its evolution process from signal theory (and wavelet theory) perspective. We have found some regularity in the volatility evolution process. The minimum total distance indicator between Fourier coefficients is proposed as a measure of such regularity. This indicator has a nature of volatility lower bound.**

Keywords **– Continuous wavelet transform, Fourier transform, helical (spiral) structure, minimum total distance, stock indices, time series, volatility, wavelet analysis.**

I. Introduction

This paper continues the series of papers [2]–[5] devoted to the volatility analysis of financial time series – stock market data. The idea of this research is to discover new properties of financial time series using wavelet decomposition.

The main key stone of this research is a vector representation of functions; this idea is widely used in the Fourier transform, where a main function is represented as a sum of elementary functions. The same idea is used in a wavelet transform, where the compact defined function, shifted and scaled, is used as a mother wavelet function, and any signal can be represented as a sum of mother wavelet functions.

In the financial time series analysis, it is important not only to analyze the financial time series (as the signal or stochastic process), but also the volatility indicator (amount of variation or dispersion from the average). Volatility indicator is evolving in time and volatility forecast is very important in the financial time series analysis, since the price of derivative financial instruments is volatility dependent.

The main approaches used in the current paper are as follows:

1. The signal (financial time series) can be decomposed in parts or components by using wavelet filtration, and each component can be analyzed from a volatility perspective. The sum of volatility of all components is proportional to overall signal volatility.

2. The signal can be considered from a volatility perspective and evolving volatility can be considered a signal, which can be decomposed in components by using wavelet filtration.

Volatility components, which are evolving in time, discover some specific properties of the signal, which cannot be discovered in an original signal without decomposition. Since wavelet decomposition uses the shifted and scaled mother wavelet function, it acts like a microscope, highlighting certain parts of the signal. These specific properties were described in [1]–[4] with the author term “North-East Volatility Wind Effect”.

The main conclusion made in paper [2] is the following – a slight increase in volatility in the low-frequency components of the signal leads to significant disturbances in high-frequency components that destine the entire signal volatility growth. This effect is called “North-East Volatility Wind”. This conclusion is important for risk management on the stock markets. “North-East Volatility Wind” Effect described in [2] brings out a deeper understanding of volatility evolution and an opportunity to illuminate most dramatically market drawdowns initially. This opportunity is explained by an ability to see very small changes in volatility (logarithmic variance) of the low-frequency components of the signal.

Further research of volatility evolution in components (volatility layers) and volatility transmission between layers is conducted in [3].

However, volatility in the perspective of scaling parameter can also be considered the signal (defined in a space of scaling parameter), which has a regular wave form. This signal is named volatility wave, which is the object of current research. Evolving volatility wave is represented by Fourier image by using discrete Fourier transform. Here and further the research algorithm is described.

II. Research Algorithm

Research algorithm consists of 4 (four) blocks: Data Loading & Preprocessing Block, Signal Decomposition Block, Volatility Analysis Block and Fourier Analysis Block. Research algorithm is described in Fig. 1. In the output of each block, output is illustrated in figures – by 2D and 3D plots. Number of each plot is related to the figure number in the current paper.

A. Data Loading and Preprocessing Block

Data Loading & Preprocessing Block uses the ticker (finance.yahoo.com data depositary ticker as id) and provides the analyzed signal $X(t)$ in output. This block consists of 3 main parts: data loader operator, which downloads daily financial

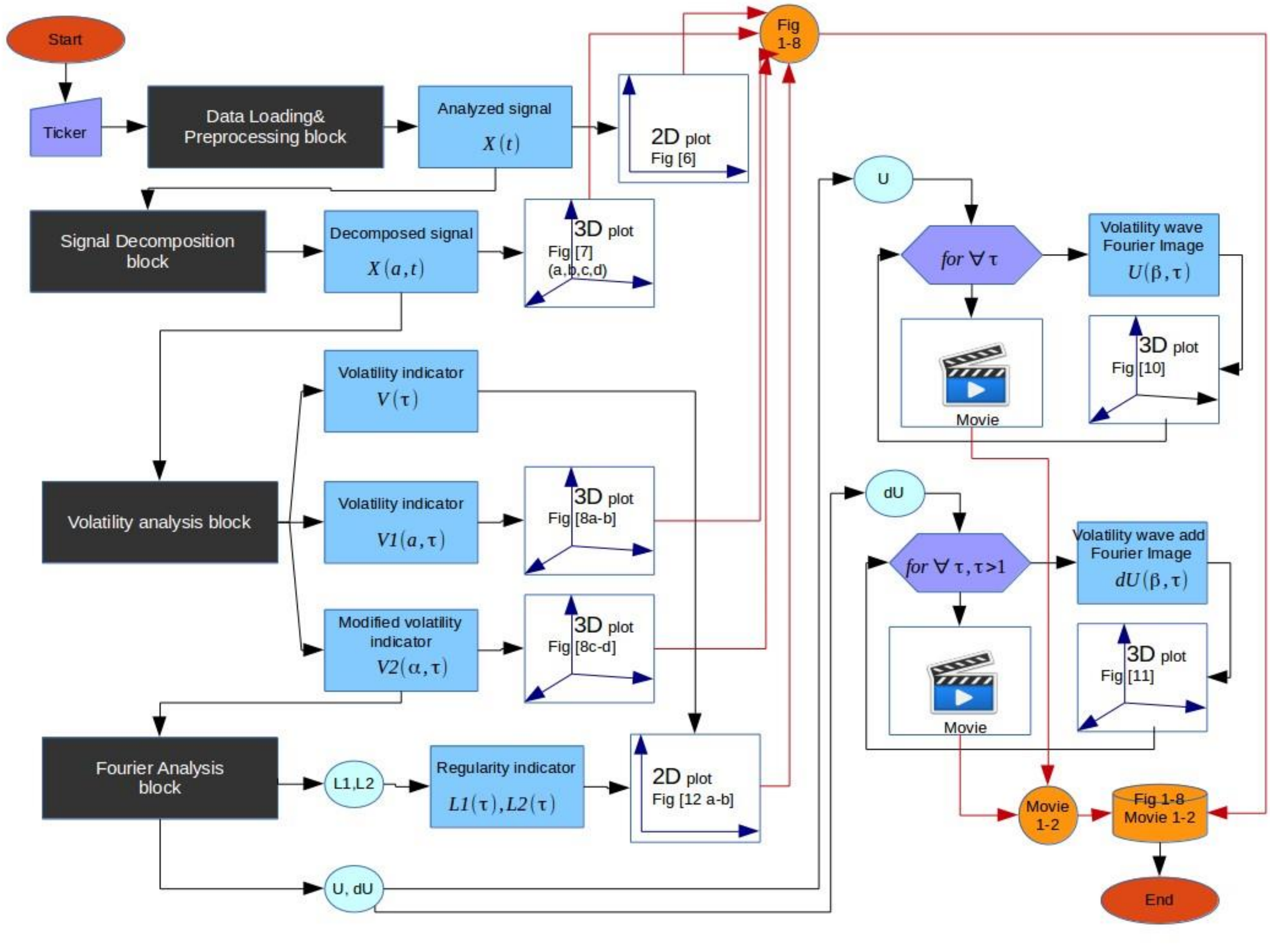


Fig. 1. Research algorithm, block-scheme.

time series (for the whole available period) by a predefined ticker. This operator fits Log-operand, which calculates a natural logarithm of financial time series in order to remove the network effect in financial time series data. Log-operands fit Norm-operand, which is normalizing additions of signal to $N(0,1)$ process, in other words, it is calculating additions to signal $\Delta X(t)$ and normalizing it to the process with zero mathematical expectation $\mu = 0$ and unit standard deviation $\sigma = 1$. Norm-operand provides output of Data Loading & Preprocessing Block, which is called analyzed signal $X(t)$. The algorithm is described in Fig. 2.

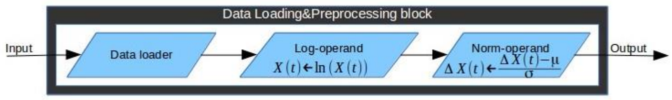


Fig. 2. Data Loading and Preprocessing Block.

Procedure of this block was described in [2]–[4]. Programming codes in Matlab language are shown in [5].

B. Signal Decomposition Block

Signal Decomposition Block is fitted by Data Loading & Preprocessing Block. Signal Decomposition Block is placed between Data Mining and Preprocessing Block and Signal Analysis Block.

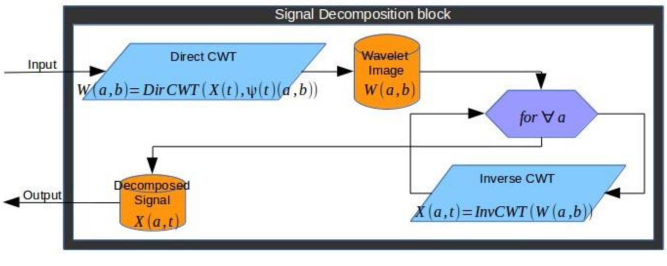


Fig. 3. Signal Decomposition Block.

The block consists of two parts: Direct Continuous Wavelet Transform (Direct CWT) and Inverse Continuous Wavelet Transform (Inverse CWT) operands. Direct CWT operand provides Wavelet image $W(a,b)$ of the analyzed signal $X(t)$. Inverse CWT operand loads Wavelet image $W(a,b)$ for each scaling parameter a and transforms it back to a part of analyzed signal $X(a,t)$. Inverse CWT operand operates in loops $(\forall a)$ (for each scaling parameter a), as a result by using Direct CWT and Inverse CWT operands analyzed signal $X(t)$ is decomposed to $X(a,t)$ [2]. Signal decomposition by using wavelet transform keeps the following idea: the number of signal components only depends on scaling parameter a and by summing all components of decomposed signal original (analyzed) $X(t)$ signal can be reconstructed. Decomposition of analyzed signal $X(t)$ to $X(a,t)$ serves for the subsequent volatility analysis, which has been done for $X(a,t)$. Subsequent volatility analysis is performed for the decomposed signal, which describes volatility evolution since and of volatility transmission. Exploration of Signal Decomposition Block was described in [2]–[4]. Programming codes in Matlab language are shown in [5].

C. Volatility Analysis Block

Volatility Analysis Block is fed by Signal Decomposition Block output – decomposed signal $X(a,t)$. Volatility Analysis Block run in loop for each scaling parameter a. By selecting decomposed signal $X(a,t)$ for defined a the block by using a window function selects 25-trading-day interval τ in time space t (in other words from $X(a,t)$ for certain a it selects $t \in [1:25]$) and calculates logarithmic variance $\ln(D(X(a,\tau)))$ (or volatility indicator $V1(a,\tau)$) [2].

After this window function is shifted in time till the last τ and the procedure is repeated for each a scaling parameter $\forall a$.

Volatility indicator $V1$ is stored in memory and further shown in a 3D plot (in Fig. 8), as well as it is used for modified volatility indicator calculation.

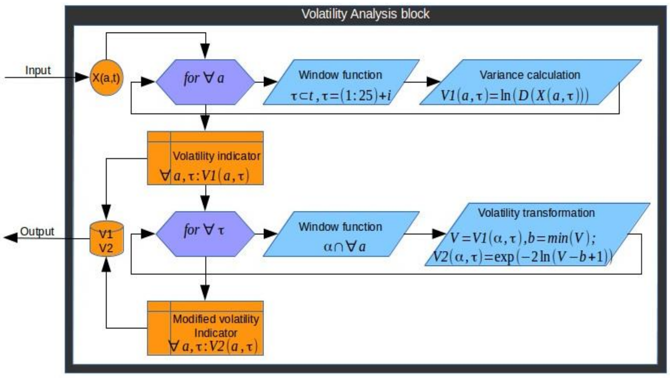


Fig. 4. Volatility Analysis Block.

Modified volatility indicator calculation is based on volatility indicator $V1$ data and starts from running τ index (from one to last τ). For certain τ volatility indicator $V1(a,\tau)$ is selected along a horizon $V1(:,\tau)$ (or in other words $V1(\alpha,\tau), \alpha \in [a_{\min}, a_{\max}]$). Here and further $V1(\alpha,\tau)$ is abbreviated as $V1$ volatility wave. After this, specific transformation with $V1(\alpha,\tau)$ is done in accordance with equation (1). After this the procedure is repeated by changing $\tau \leftarrow \tau+1$ to the last τ.

$$V2(\alpha,\tau) = \exp(-2\ln(V-b+1)),$$
$$V = V1(\alpha,\tau), \alpha \in [a_{\min}, a_{\max}], b = \min(V). \quad (1)$$

Illustration of this transformation is done in Fig. 9. This modified volatility indicator $V2$ by its nature illuminates local maxima lines of $V1$.

Schematic illustration of Volatility Analysis Block is performed in Fig. 4. This block in the output provides $V1$ and $V2$ volatility indicators, which are illustrated in 3D plots in Fig. 9. Modified volatility indicator $V2$ is used for the subsequent analysis in Fourier Analysis Block.

Detailed exploration of volatility indicator $V1$ calculation can be found in [2]–[3]. Programing code in Matlab environment is provided in [5]. Transformation from $V1$ to $V2$ is described in [6].

D. Fourier Analysis Block

Fourier Analysis Block uses modified volatility indicator $V2$ as input and provides several outputs: Fourier image of volatility wave $U(\beta,\tau)$, Fourier image of volatility wave growth $dU(\beta,\tau)$, regularity indicators $L1(\tau)$ (calculated from $U(\beta,\tau)$) and $L2(\tau)$ (calculated from $dU(\beta,\tau)$). Regularity indicators $L1(\tau)$ and $L2(\tau)$ are a measure of minimum total distance between all Fourier image coefficients. Here and further a detailed explanation is provided.

Fourier Analysis Block uses modified volatility indicator $V2$ as input. It runs a running index τ and for certain τ selects volatility wave $V2(:,\tau)$ or $V2(\alpha,\tau), \alpha \in [a_{\min}, a_{\max}]$. This volatility wave is transformed from time to frequency domain by using discrete Fourier transform algorithm, described in (2).

$$U(\beta,\tau) = \left\langle V2(\alpha,\tau), e^{j\beta\alpha} \right\rangle. \quad (2)$$

As a result, Fourier image of volatility wave $U(\beta,\tau)$ (where β is a period). Fourier analysis is repeated for each τ. The same procedure is done for volatility wave growth $dV2$. The $dV2$ indicator is calculated by (3).

$$dV2(\alpha,\tau) = V2(\alpha,\tau) - V2(\alpha,\tau-1). \quad (3)$$

Volatility wave growth representation in frequency domain is done by a fast Fourier transform, as a result Fourier image of volatility wave growth $dU(\beta,\tau)$ is calculated in the block.

Fourier images $U(\beta,\tau)$ and $dU(\beta,\tau)$ have specific properties – Fourier image coefficients demonstrate a clear regular helical (spiral) structure. Fourier images of volatility waves calculated for stock indices are shown in Figs. 10–11.

Analogical results for Fourier analysis of volatility evolution waves for DJIA (The Dow Jones Industrial Index) are shown in papers [6]–[7].

Authors propose Fourier image regularity indicators $L1(\tau)$ and $L2(\tau)$ defined on time space τ according equation (4):

$$L1(\tau) = -\sum_{\forall\beta} |U(\beta,\tau) - U(\beta-1,\tau),|$$
$$L2(\tau) = -\sum_{\forall\beta} |dU(\beta,\tau) - dU(\beta-1,\tau),| \quad (4)$$

The block-scheme of Fourier Analysis Block is shown in Fig. 5.

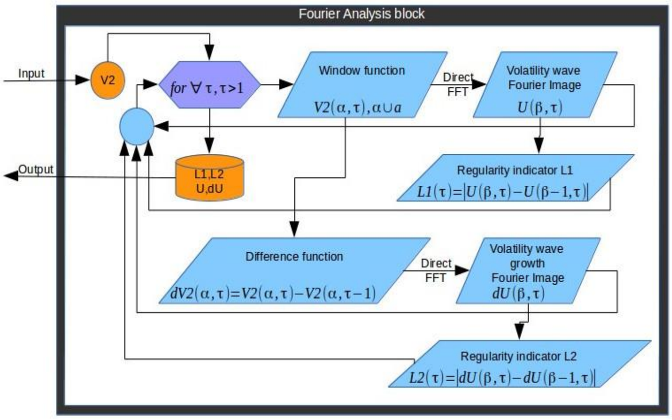


Fig. 5. Fourier Analysis Block.

According to the block-scheme in Fig. 5, the block runs in for loops with running index τ and calculates $U(\beta,\tau),dU(\beta,\tau)$ $U(\beta,\tau),dU(\beta,\tau)$ Fourier images and from them $L1(\tau)$ and $L2(\tau)$ regularity indicators. As a result, the block provides output with $L1, L2$ which are loaded to 2D plots.

By running for loops with running index τ, U,dU Fourier images are loaded to scatter 3D plots and the result image is added to movie, stored in a hard drive. Detailed illustrations of movie making and plotting are shown in Fig. 1.

III. Financial Time Series Analysis

Here and further, for the subsequent research the Dow Jones Industrial Index financial time series data of the period (1993 – nowadays) are selected.

By feeding Data Loading & Preprocessing Block with ticker (^DJI), the block provides in output the signal shown in Fig. 6.

In the Signal Decomposition Block, the analyzed signal is decomposed in parts by using wavelet filtration – Direct CWT and Inverse CWT. The output is provided from various viewpoints in Fig. 7.

Decomposed signal is used as input for Volatility Analysis Block, which provides a picture of volatility evolution shown in Fig. 8.

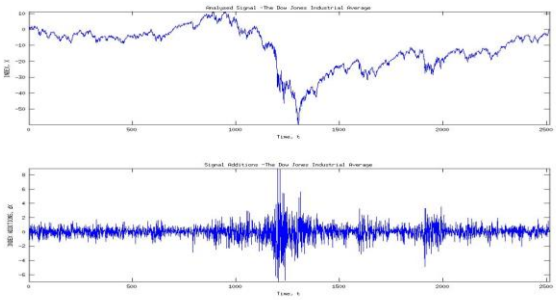


Fig. 6. Analyzed signal of the 2Dow Jones industrial average (above – the signal, below – signal growth).

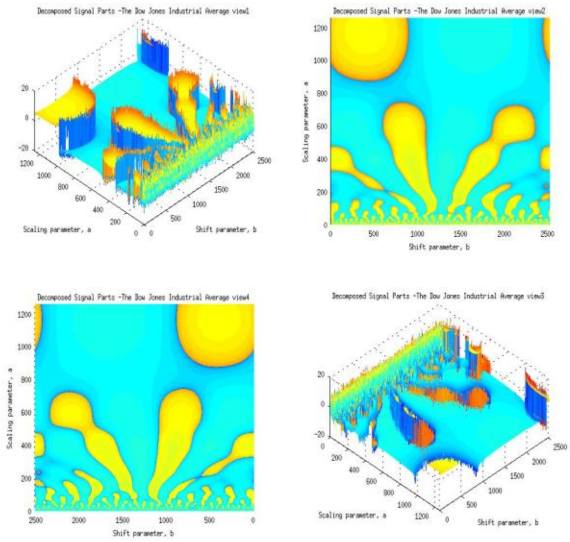


Fig. 7. Decomposed signal of the Dow Jones industrial average.

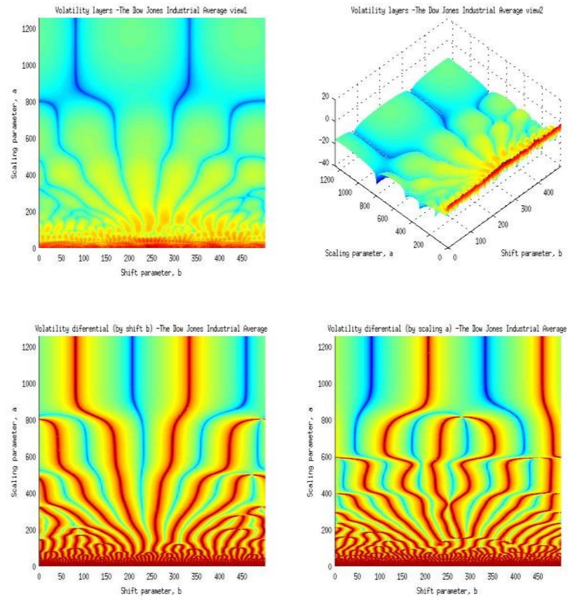


Fig. 8. Volatility analysis of the Dow Jones industrial average.

For the subsequent analysis, the modified volatility data $V2$ are used. The difference between Volatility $V1$ and Modified volatility $V2$ is shown in Fig. 9. Modified Volatility $V2$ illuminates local maxima lines.

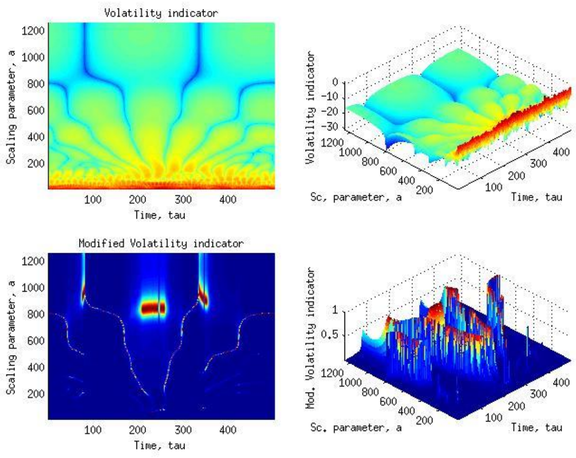
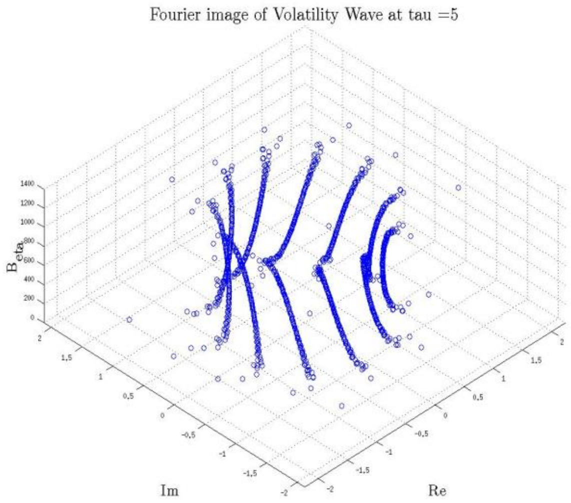
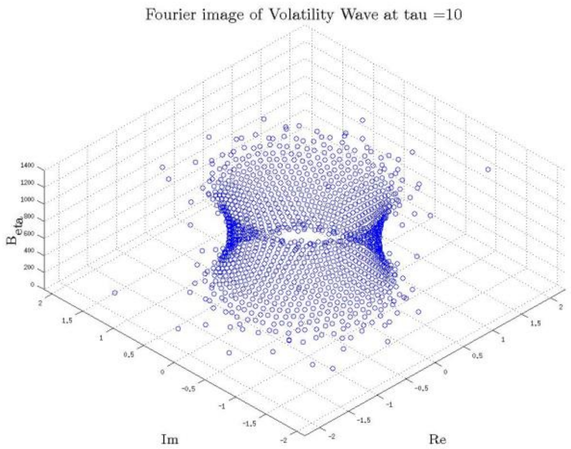


Fig. 9. Volatility (top) and modified volatility (bottom) indicators.

For the subsequent analysis, the modified volatility data $V2$ fits Fourier Analysis Block, which analyzes volatility waves in a frequency domain. Fourier images of volatility wave $U(\beta,\tau)$ are shown in Fig. 10 (a-c).





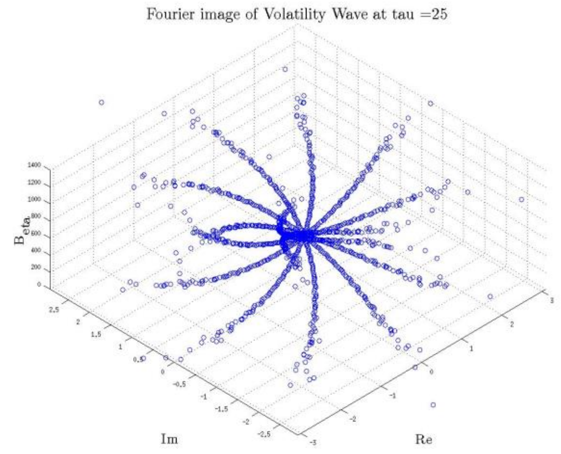
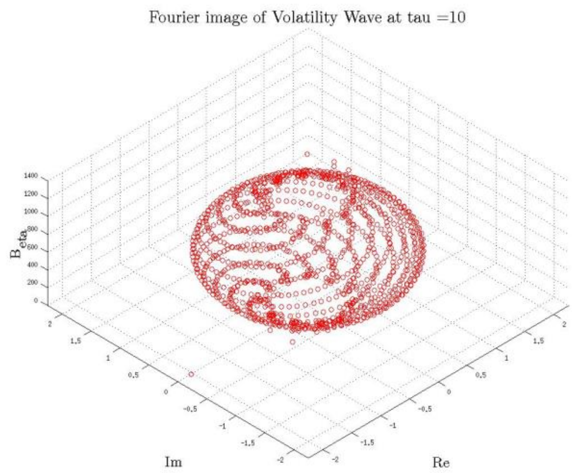
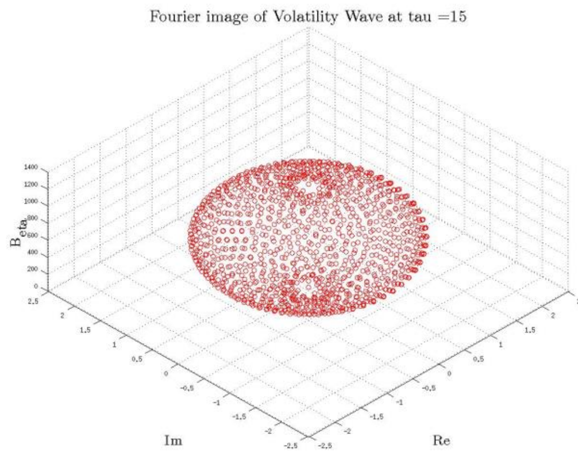
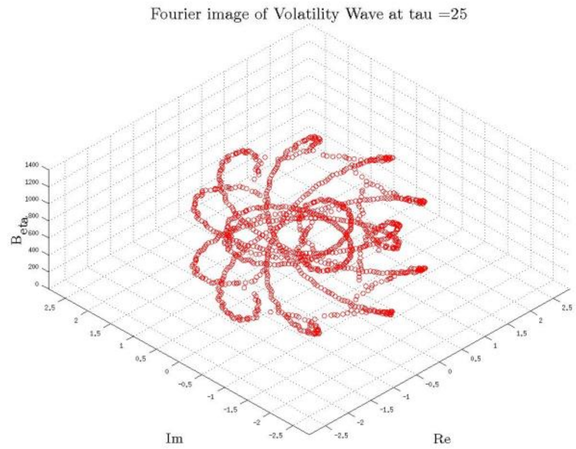


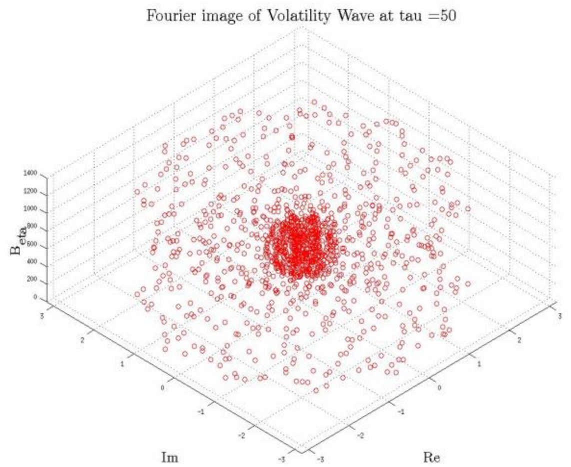
Fig. 10. a-c) Fourier image of volatility wave in time $\tau = \{5,10,25\}$.

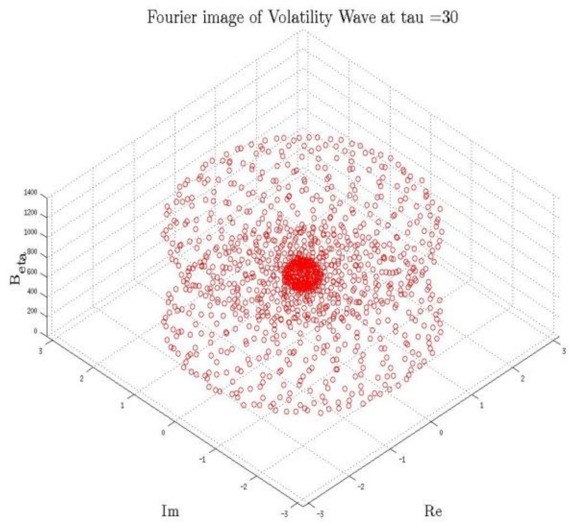
Fourier images of volatility wave growth $dU(\beta,\tau)$ are shown in Fig. 11 (a-f).











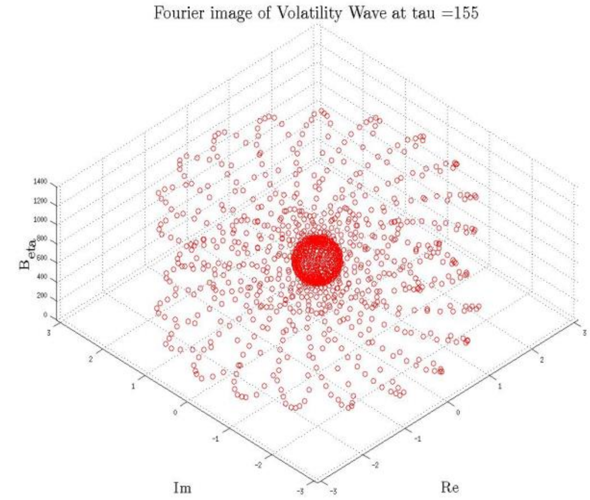


Fig. 11. a-f) Fourier image of volatility wave growth in time $\tau = \{10,15,25,30,30,155\}$.

Evolving in time τ Fourier images of both $U(\beta,\tau), dU(\beta,\tau)$ are saved together with a volatility indicator in the prepared movie. According to Figs. 10–11, the Fourier images $U(\beta,\tau), dU(\beta,\tau)$ show a clear helical (spiral) structure.

Fourier images $U(\beta,\tau), dU(\beta,\tau)$ evolve in time τ and show some degree of regularity at each τ . The measure of regularity described by indicators $L1, L2$ measuring minimum total distance of scatter plot points is shown in Fig. 12. Indicators $L1, L2$ are shown together with overall volatility.

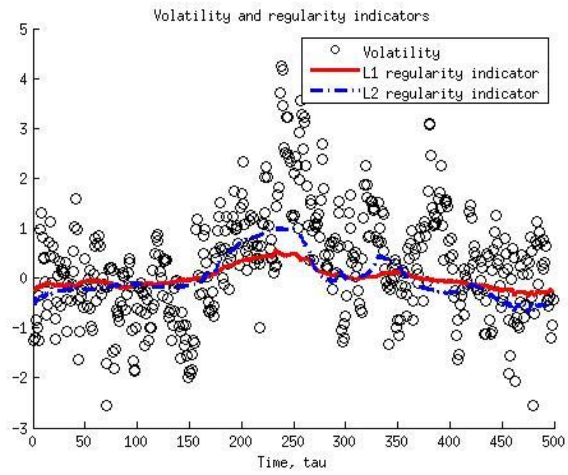


Fig. 12. a) $L1$ and $L2$ indicators, overall volatility indicator.

As it is shown in a plot, regularity indicators $L1, L2$ approximate a volatility trend. In order to check this, by using wavelet filtration (discrete wavelet transform decomposition till the 10th level with Dobeshi wavelet "db7") high frequency components of volatility indicator were removed. The result is shown in Fig. 12.

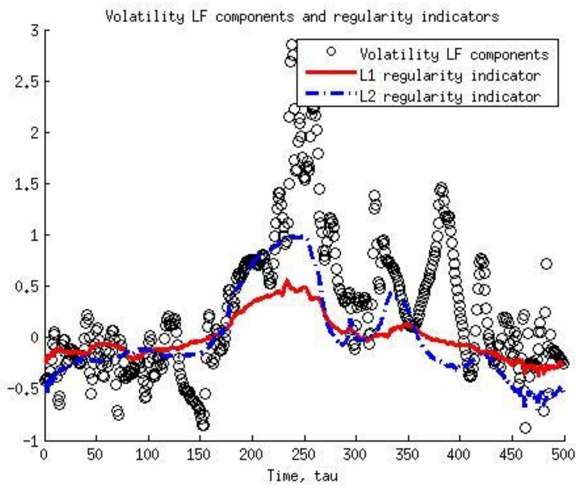


Fig. 12. b) $L1$ and $L2$ indicators, LF components of overall volatility indicator.

Regularity indicators $L1, L2$ have high correlation with volatility indicator low frequency components. The Pearson correlation results are shown in Table I.

TABLE I

Pearson Cross-correlation Matrix

	Volatility indicator	L1 indicator	L2 indicator
Volatility indicator	1	0.7443	0.7145
*L*1 indicator	0.7443	1	0.9665
*L*2 indicator	0.7145	0.9665	1

In order to bring clearness to the analyzed process – the process of wavelet filtration, evolving volatility measure and Fourier analysis of the so-called volatility wave – the process and output of this process are to be described next.

IV. Research Results and Discussion

Suppose that each investor reacts to the events happened on financial markets by making their individual decisions taking into account their own investment horizon. Moreover, this decision is based on filtering the time series of stock prices.

If the situation is stable in the market, individual investors will not change their open positions rapidly, otherwise market instability and price volatility should rise. By itself volatility is a measure of strikes between individual investors. More strikes, more jagged the index is.

Understanding of the so-called volatility wave starts with understanding of interrelations between various investment horizon investors. Local maxima of volatility wave dedicate an investment horizon, where the strike between investors is more noticeable.

Understanding of Fourier image of volatility wave is similar. Maximum magnitude Fourier harmonics should indicate frequency (which is inverse to an investment horizon), which responds to volatility wave local maxima.

The second important understanding of Fourier image of volatility wave is illumination of structure. As it is shown in Fig. 10 and Fig. 11, markets definitely have a structure, since various investment horizon investors should interrelate to each other. This structure is evolving in time, can take different forms. Increase of regularity or emanation of certain harmonics normally means running a volatility wave, which results in the market crisis. This statement becomes clear, understanding $L1, L2$ measures, while the total distance between all Fourier coefficients turns to minimum, a volatility indicator trend (low frequency component) rises.

Discovering the structure of volatility wave Fourier image is very important, currently the minimum total distance $L1, L2$ indicators need modification to describe different regularity structures.

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Andrejs Pučkovs, Andrejs Matvejevs. Akciju tirgu struktūras izmaiņu atklāšana, izpētot volatilitātes viļņu Furjē attēla spirālveida struktūru
Šajā rakstā tiek aplūkoti alternatīvie volatilitātes rādītāji, kas balstīti uz volatilitātes viļņa Furje attēla spirālveida regularitātes mērījumiem. Volatilitātes vilnis tiek iegūts ar veivlet filtrāciju (ar tiešo un apgriezto nepārtraukto viļņu pārveidojumiem), veicot analizējamā signāla (finanšu laikrindu) dekompozīciju ar turpmāko volatilitātes (jeb logaritmiskās dispersijas) pētījumu katrā signālu komponentē. Turpmāk volatilitātes rādītājs tiek pārveidots noteiktā veidā, tā kā ir aprakstīts šai rakstā. Šī pārveidojuma jēdziens ir volatilitātes indikatora mērogošana un lokālo maksimumu izšķiršana. Modificēts volatilitātes rādītājs tiek analizēts mērogošanas rādītāja griezumā katrā laika momentā „tau". Modificēts volatilitātes rādītājs mērogošanas parametra griezumā veido viļņveida formu, dēvētu par volatilitātes vilni. Ar Furjē analīzi volatilitātes vilnis tiek pārveidots Furjē attēlā. Rezultātā reālās un imaginārās Furjē attēla daļas veido regulārās formas spirālveida struktūru. Furjē attēla regularitāte tiek noteikta ar $L1$ un $L2$ rādītājiem, kas tiek aprēķināti, kā minimālā distance starp visiem volatilitātes viļņa Furjē attēla koeficientiem. $L1$ un $L2$ rādītāji pēc savas dabas ir alternatīvi volatilitātes rādītāji. Šis raksts atklāj jaunu skatu uz volatilitāti un to evolūciju Furjē attēla koeficientu spirālveida struktūras griezumā. Šī pieeja ļauj prognozēt jaunas finanšu krīzes rašanos**.**

Андрей Пучков, Андрей Матвеев. Выявление структурных изменений фондовых рынков, исследующих спиралевидную структуру Фурье образа волн волатильности
В данной статье рассматриваются альтернативные оценки волатильности, основанные на оценке регулярности спиралевидной структуры Фурье образа волн волатильности. Волна волатильности, получаемая с помощью вейвлет-декомпозиции (Прямого Непрерывного вейвлет-преобразования и Обратного Непрерывного вейвлет-преобразования), посредством декомпозиции анализируемого сигнала (финансового временного ряда) с последующим вычислением показателя волатильности (логарифмической дисперсии) для каждого компонента сигнала. Далее показатель волатильности преобразуется согласно преобразованию, описанному в настоящей работе. Суть данного преобразования заключается в масштабировании и выделении линий локальных максимумов для показателя волатильности. В результате преобразования получаем модифицированный показатель волатильности. Модифицированный показатель волатильности анализируется в разрезе показателя масштаба для каждого временного показателя «тау». Модифицированный показатель волатильности в разрезе показателя масштаба образует волнообразную форму, названную волной волатильности. Посредством Фурье анализа волны волатильности выявляется Фурье образ волны волатильности. В результате мнимые и действительные части коэффициентов Фурье образуют четкую спиралевидную структуру регулярной формы. Регулярность или структурность Фурье образа определяется с помощью показателей $L1$, $L2$, рассчитываемых как минимальное расстояние между всеми коэффициентами Фурье образа волны волатильности. Показатели регулярности $L1$, $L2$ являются альтернативными показателями волатильности. Данная статья открывает новое видение волатильности и её эволюции с точки зрения спиралевидной структуры коэффициентов Фурье образа волны волатильности. Данное видение позволяет открыть новые способы предсказания финансовых кризисов.