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Algorithm for Monitoring Minimum Cost in Fuzzy Dynamic Networks

Alexander Bozhenyuk[1], Evgeniya Gerasimenko[2], [1-2] *Southern Federal University*

***Abstract* – The present paper examines the task of minimum cost flow finding in a fuzzy dynamic network with lower flow bounds. The distinguishing feature of this problem statement lies in the fuzzy nature of the network parameters, such as flow bounds, transmission costs and transit times. The arcs of the considered network have lower bounds. Another feature of this task is that fuzzy flow bounds, costs and transit times can vary depending on the flow departure time. Algorithm, which implements the solution of considered problem, is proposed.**

***Keywords* – Fuzzy dynamic network, lower flow bounds, minimum cost flow**

I. Introduction

Conventional tasks of finding a maximum flow and minimum cost flow assume that the instant flow passes along the arcs of the graph that certainly is simplification of the real life. Such tasks are called static flow tasks. In fact, it turns out that the flow spends certain time passing along the arcs of the graph. Then, we turn to dynamic networks, in which each flow unit passes from the source to the sink for a period of time less than given. Dynamic network is a network $G = (X, A)$, where $X = \{x_1, x_2 ..., x_n\}$ – the set of nodes, $A = \{(x_i, x_j)\}, i, j \in I = \overline{1,n}$ – the set of arcs. Each arc of the dynamic graph (x_i, x_j) is denoted by two parameters: transit time τ_{ij} and arc capacity u_{ij}. The time horizon $T = \{0,1,..., p\}$ determining that all flow units sent from the source must arrive at the sink within time p is given [1].

Dynamic networks describe complex systems, problems of decision-making, models, whose parameters can vary over time. Such models can be found in communication systems, economic planning, transportation systems and many other applications, so they have a wide range of practical applications.

II. Literature Review

Historically, the maximum flow finding in dynamic graphs was the first task in dynamic graphs, described in the literature. The notion "dynamic flow" was proposed by Ford and Fulkerson [2] as a task of maximum dynamic flow finding in a network. This problem is related to finding a maximum flow, passing from the source (s) to the sink (t), $s, t \in X$ in the network for p discrete time periods, starting from zero period of time.

The task of minimum cost flow finding in dynamic graphs is that of searching for flows of the given value, which have a minimum cost in dynamic graphs. This field, which appeared later, is a more complex sphere of investigations. Fleischer and Skutella [3] examined this problem. Cai et al. [4], Halpern [5] considered networks with transit parameters. The subproblem of the minimum cost flow finding in dynamic graphs is the shortest path problem. This problem was introduced by Cooke and Halsey [6] and was widely reported in the literature by such authors as Ahuja et al. [7], Pallottino and Scutella [8] in terms of nonnegative transit times.

The fact that the flow, passing along the arcs of the graph, can have lower bounds usually is not taken into account in the literature. For example, a network that consists of railways, sea and air roads is considered. Therefore, the freight trains have a certain level of load, which exceeds a profitability threshold; transport planes do not fly at a low load. Thus, it is necessary to introduce lower flow bounds, which can lead to the absence of feasible flow.

III. Problem Statement

The task of minimum cost flow finding in a fuzzy dynamic network with fuzzy lower and upper flow bounds has the following problem statement:

$$Minimize \sum_{\theta=0}^{p} \sum_{(x_i,x_j)\in\tilde{A}} \tilde{c}_{ij}(\theta) \times \tilde{\xi}_{ij}(\theta), \quad (1)$$

$$\sum_{\theta=0}^{p} \sum_{x_j \in X} [\tilde{\xi}_{sj}(\theta) - \tilde{\xi}_{js}(\theta - \tau_{js}(\theta))] - \tilde{\rho}(p) = \tilde{0}, \quad (2)$$

$$\sum_{x_j \in X} [\tilde{\xi}_{ij}(\theta) - \tilde{\xi}_{ji}(\theta - \tau_{ji}(\theta))] = \tilde{0},\ x_i \neq s,t;\ \theta \in T, \quad (3)$$

$$\sum_{\theta=0}^{p} \sum_{x_j \in X} [\tilde{\xi}_{tj}(\theta) - \tilde{\xi}_{jt}(\theta - \tau_{jt}(\theta))] + \tilde{\rho}(p) = \tilde{0}, \quad (4)$$

$$\tilde{l}_{ij}(\theta) \le \tilde{\xi}_{ij}(\theta) \le \tilde{u}_{ij}(\theta),\ for\ \theta : \theta + \tau_{ij}(\theta) \le p, \theta \in T. \quad (5)$$

Equation (1) means that it is necessary to find the minimum cost transmission route of the given flow value for the specified number of time periods. Equation (2) indicates that the given flow value $\tilde{\rho}$ for p time periods is equal to the flow, leaving the source for p time periods $\sum_{\theta=0}^{p}\tilde{\xi}_{sj}(\theta)$. Equation (4) reflects that the given flow value $\tilde{\rho}$ for p time periods is equal to the flow entering the sink for p time periods $\sum_{\theta=0}^{p}\tilde{\xi}_{jt}(\theta-\tau_{jt})$. The amount of flow entering the source $\sum_{\theta=0}^{p}\tilde{\xi}_{js}(\theta-\tau_{js})$ for p time periods is equal to the flow $\sum_{\theta=0}^{p}\tilde{\xi}_{tj}(\theta)$ leaving the sink for p time periods and is equal to $\tilde{0}$. For each node x_i except the source and the sink and for each time period θ the amount of flow $\tilde{\xi}_{ji}(\theta-\tau_{ji})$ entering x_i at each period of time $(\theta-\tau_{ji})$ is equal to the amount of flow $\tilde{\xi}_{ij}(\theta)$ leaving x_i at time θ as stated in (3). Inequality (5) indicates that the flows $\tilde{\xi}_{ij}(\theta)$ for time periods $\theta:\theta+\tau_{ij}(\theta)\le p, \theta\in T$ should be more than lower flow bounds $\tilde{l}_{ij}(\theta)$ and less than upper flow bounds $\tilde{u}_{ij}(\theta)$ along the corresponding arcs.

In other words, it is necessary to carry $\tilde{\rho}(p)$ flow units with minimum cost in a dynamic network taking into account lower flow bounds in such a way that the last flow unit would enter the sink at time period not later than *p*. In this case, upper flow bounds, lower flow bounds and transmission costs are transit.

We represent the formal algorithm describing the solution to the problem of finding a minimum cost dynamic flow with upper and lower fuzzy flow bounds in a fuzzy transportation network with time-varying fuzzy flow bounds, transmission costs and time-varying crisp flow transit times along the arcs.

Step 1. Go to the time-expanded fuzzy static graph $\tilde{G}_p$ from the given fuzzy dynamic graph $\tilde{G}$ by expanding the original dynamic graph in the time dimension by making a separate copy of every node $x_i \in X$ at every time $\theta\in T$. Let $\tilde{G}_p=(X_p,\tilde{A}_p)$ represent a fuzzy time-expanded static graph of the original dynamic fuzzy graph. The set of nodes X_p of the graph $\tilde{G}_p$ is defined as $X_p=\{(x_i,\theta):(x_i,\theta)\in X\times T\}$. The set of arcs $\tilde{A}_p$ consists of arcs from each node-time pair $(x_i,\theta)\in X_p$ to every node-time pair $(x_j,\theta+\tau_{ij}(\theta))$, where $x_j\in\Gamma(x_i)$ and $\theta+\tau_{ij}(\theta)\le p$. Fuzzy upper flow bounds $\tilde{u}(x_i,x_j,\theta,\theta+\tau_{ij}(\theta))$ joining (x_i,θ) with $(x_j,\theta+\tau_{ij}(\theta))$ are equal to $\tilde{u}_{ij}(\theta)$ and fuzzy lower flow bounds $\tilde{l}(x_i,x_j,\theta,\theta+\tau_{ij}(\theta))$ joining (x_i,θ) with $(x_j,\theta+\tau_{ij}(\theta))$ are equal to $\tilde{l}_{ij}(\theta)$, transmission cost $\tilde{c}(x_i,x_j,\theta,\theta+\tau_{ij}(\theta))$ of one flow unit along the arc connecting the node-time pair (x_i,θ) with $(x_j,\theta+\tau_{ij}(\theta))$ is equal to $\tilde{c}_{ij}(\theta)$.

Step 2. Determine, if the time-expanded fuzzy graph $\tilde{G}_p$, corresponding to the initial dynamic graph $\tilde{G}$, has a feasible flow. Introduce the artificial source s^* and sink t^* in the graph $\tilde{G}_p$ and turn to the graph $\tilde{G}_p^*=(X_p^*,\tilde{A}_p^*)$ without lower flow bounds according to the method, described in [12]. The set X_p^* consists of the nodes from the set X_p and the artificial nodes s^* and t^*. Introduce the arcs, connecting the node-time pair $(t,\forall\theta\in \mathrm{T})$ and $(s,\forall\theta\in \mathrm{T})$ with upper fuzzy flow bound $\tilde{u}^*(t,s,\forall\theta\in T,\forall\theta\in T)=\infty$, lower fuzzy flow bound $\tilde{l}^*(t,s,\forall\theta\in T,\forall\theta\in T)=\tilde{0}$, transmission cost $\tilde{c}^*(t,s,\forall\theta\in T,\forall\theta\in T)=\tilde{0}$ in the graph $\tilde{G}_p^*$. It means that every node t in each time period from p is connected with every node s at all time periods in the graph $\tilde{G}_p^*$. Introduce the following modification for each arc connecting the node-time pair (x_i,ϑ) with the node-time pair $(x_j,\theta=\vartheta+\tau_{ij}(\vartheta))$ with nonzero lower fuzzy flow bound $\tilde{l}(x_i,x_j,\vartheta,\theta)\ne\tilde{0}$: 1) reduce $\tilde{u}(x_i,x_j,\vartheta,\theta)$ to $\tilde{u}^*(x_i,x_j,\vartheta,\theta)=\tilde{u}(x_i,x_j,\vartheta,\theta)-\tilde{l}(x_i,x_j,\vartheta,\theta)$, $\tilde{l}(x_i,x_j,\vartheta,\theta)$ to $\tilde{0}$, $\tilde{c}^*(x_i,x_j,\vartheta,\theta)=\tilde{c}(x_i,x_j,\vartheta,\theta)$; 2) introduce the arcs connecting s^* with (x_j,θ), and the arcs connecting t^* with (x_i,ϑ) with upper fuzzy flow bounds equal to lower fuzzy flow bounds $\tilde{u}^*_{s^*x_j(\theta)}=\tilde{u}^*_{x_i(\vartheta)t^*}=\tilde{l}(x_i,x_j,\vartheta,\theta)$, zero lower fuzzy flow bounds $\tilde{l}^*_{s^*x_j(\theta)}=\tilde{l}^*_{x_i(\vartheta)t^*}=\tilde{0}$ and zero transmission costs $\tilde{c}^*_{s^*x_j(\theta)}=\tilde{c}^*_{x_i(\vartheta)t^*}=\tilde{0}$.

Step 3. Build a fuzzy residual network $\tilde{G}_p^{*\mu}$ depending on the flow values going along the arcs of the graph $\tilde{G}_p^*$. Fuzzy residual network $\tilde{G}_p^{*\mu}=(X_p^{*\mu},\tilde{A}_p^{*\mu})$ is constructed according to the time-expanded fuzzy static graph $\tilde{G}_p^*$ without lower fuzzy flow bounds depending on the flow values $\tilde{\xi}^*(x_i,x_j,\vartheta,\theta)$ going along it as follows: each arc in the fuzzy residual network $\tilde{G}_p^{*\mu}$, connecting the node-time pair (x_i^μ,ϑ) with the node-time pair (x_j^μ,θ), whose flow $\tilde{\xi}^*(x_i,x_j,\vartheta,\theta)$ is sent along at each period of time $\vartheta\in T$ has fuzzy residual capacity $\tilde{u}^{*\mu}(x_i,x_j,\vartheta,\theta)=\tilde{u}^*(x_i,x_j,\vartheta,\theta)-\tilde{\xi}^*(x_i,x_j,\vartheta,\theta)$ with transit time $\tau^{*\mu}(x_i,x_j,\vartheta,\theta)=\tau^*(x_i,x_j,\vartheta,\theta)$ and modified transmission cost $\tilde{c}^{*\mu}(x_i,x_j,\vartheta,\theta)=\tilde{c}^*(x_i,x_j,\vartheta,\theta)$, and a reverse arc connecting the node-time pair (x_j^μ,θ) with

(x_i^{μ},ϑ) with residual fuzzy arc capacity $\widetilde{u}^{*\mu}(x_j,x_i,\theta,\vartheta)=\widetilde{\xi}^*(x_i,x_j,\vartheta,\theta)$ and transit time $\tau^{*\mu}(x_j,x_i,\theta,\vartheta)=-\tau^*(x_i,x_j,\vartheta,\theta)$ and modified transmission cost $\widetilde{c}^{*\mu}(x_j,x_i,\theta,\vartheta)=-\widetilde{c}^*(x_i,x_j,\vartheta,\theta)$.

Step 4. Search for the augmenting minimum cost path $\widetilde{P}_p^{*\mu}$ from the artificial source s^* to the artificial sink t^* in the constructed fuzzy residual network according to the Bellman–Ford algorithm [13].

(I) Go to **step 5** if the augmenting path $\widetilde{P}_p^{*\mu}$ is found.

(II) The flow value $\widetilde{\phi}^* < \sum_{\widetilde{l}(x_i,x_j,\vartheta,\theta)\neq\widetilde{0}} \widetilde{l}(x_i,x_j,\vartheta,\theta)$ is obtained, which is the maximum flow in $\widetilde{G}_p^*$, if the path is not found. It means that it is impossible to pass any unit of flow, but not all the artificial arcs are saturated. Therefore, the time-expanded graph $\widetilde{G}_p$ has no feasible flow as the initial dynamic fuzzy graph $\widetilde{G}$ and the task has no solution. Exit.

Step 5. Pass the minimum from the arc capacities $\widetilde{\delta}_p^{*\mu}=\min[\widetilde{u}^{*\mu}(x_i,x_j,\vartheta,\theta)]$, $(x_i,x_j)\in\widetilde{P}_p^{*\mu}$, included in the path of minimum cost $\widetilde{P}_p^{*\mu}$ along this path.

Step 6. Update the fuzzy flow values in the graph $\widetilde{G}_p^*$: replace the fuzzy flow $\widetilde{\xi}^*(x_j,x_i,\vartheta,\theta)$ along the corresponding arcs going from (x_j,ϑ) to (x_i,θ) from $\widetilde{G}_p^*$ by $\widetilde{\xi}^*(x_j,x_i,\vartheta,\theta)-\widetilde{\delta}_p^{*\mu}$ for arcs connecting node-time pair (x_i^{μ},θ) with (x_j^{μ},ϑ) in $\widetilde{G}_p^{*\mu}$ with nonpositive modified cost $\widetilde{c}^{*\mu}(x_i,x_j,\theta,\vartheta)\le 0$ and replace the fuzzy flow $\widetilde{\xi}^*(x_i,x_j,\vartheta,\theta)$ along the arcs going from (x_i,ϑ) to (x_j,θ) from $\widetilde{G}_p^*$ by $\widetilde{\xi}^*(x_i,x_j,\vartheta,\theta)+\widetilde{\delta}_p^{*\mu}$ for arcs connecting node-time pair (x_i^{μ},ϑ) with (x_j^{μ},θ) in $\widetilde{G}_p^{*\mu}$ with nonnegative modified cost $\widetilde{c}^{*\mu}(x_i,x_j,\vartheta,\theta)\ge 0$. Replace $\widetilde{\xi}^*(x_i,x_j,\vartheta,\theta)$ by $\widetilde{\xi}^*(x_i,x_j,\vartheta,\theta)+\widetilde{\delta}_p^{*\mu}\times\widetilde{P}_p^{*\mu}$.

Step 7 (I) If the flow value $\widetilde{\xi}^*(x_i,x_j,\vartheta,\theta)+\widetilde{\delta}_p^{*\mu}\times\widetilde{P}_p^{*\mu}$ of minimum cost $\widetilde{c}(\widetilde{\xi}^*(x_i,x_j,\vartheta,\theta)+\widetilde{\delta}_p^{*\mu}\times P_p^{*\mu})$ is less than $\sum_{\widetilde{l}(x_i,x_j,\vartheta,\theta)\neq\widetilde{0}} \widetilde{l}(x_i,x_j,\vartheta,\theta)$, i.e., not all artificial arcs become saturated, go to **step 3**.

(II) If the flow value $\widetilde{\xi}^*(x_i,x_j,\vartheta,\theta)+\widetilde{\delta}_p^{*\mu}\times\widetilde{P}_p^{*\mu}$ of minimum cost $\widetilde{c}(\widetilde{\xi}^*(x_i,x_j,\vartheta,\theta)+\widetilde{\delta}_p^{*\mu}\times P_p^{*\mu})$ is equal to $\sum_{\widetilde{l}(x_i,x_j,\vartheta,\theta)\neq\widetilde{0}} \widetilde{l}(x_i,x_j,\vartheta,\theta)$, i.e., all arcs from the artificial source to the artificial sink become saturated, then the value $\widetilde{\xi}^*(x_i,x_j,\vartheta,\theta)+\widetilde{\delta}_p^{*\mu}\times\widetilde{P}_p^{*\mu}$ is the required value of maximum flow $\widetilde{\sigma}^*$ of minimum cost $\widetilde{c}(\widetilde{\xi}^*(x_i,x_j,\vartheta,\theta)+\widetilde{\delta}_p^{*\mu}\times P_p^{*\mu})$. In this case the total flow along the artificial arcs connecting the node-time pairs $(t,\ \forall\,\theta\in \mathrm{T})$ with $(s,\ \forall\,\theta\in \mathrm{T})$, which is equal to $\sum_{\theta=0}^{p}\widetilde{\xi}^*(t,s,\forall\theta\in T,\forall\theta\in T)$ in $\widetilde{G}_p^*$ determines the feasible flow in time-expanded graph $\widetilde{G}_p$ with the flow value $\sum_{\theta=0}^{p}\widetilde{\xi}^*(t,s,\forall\theta\in T,\forall\theta\in T)=\widetilde{\sigma}$ of minimum cost. Turn to the graph $\widetilde{G}_p$ from the graph $\widetilde{G}_p^*$ as follows: reject artificial nodes and arcs, connecting them with other nodes. The feasible flow vector $\widetilde{\xi}=(\widetilde{\xi}(x_i,x_j,\vartheta,\theta))$ of the value $\widetilde{\sigma}$ of minimum cost is defined as follows: $\widetilde{\xi}(x_i,x_j,\vartheta,\theta)=\widetilde{\xi}^*(x_i,x_j,\vartheta,\theta)+\widetilde{l}(x_i,x_j,\vartheta,\theta)$, where $\widetilde{\xi}^*(x_i,x_j,\vartheta,\theta)$ – the flows, going along the arcs of the graph $\widetilde{G}_p^*$ after deleting all artificial nodes and connecting arcs. The network $\widetilde{G}(\widetilde{\xi})$ is obtained. Go to **step 8**.

Step 8. Construct the residual network $G(\widetilde{\xi}^{\mu}(x_i,x_j,\vartheta,\theta))$ taking into account the feasible flow vector $\widetilde{\xi}=(\widetilde{\xi}(x_i,x_j,\vartheta,\theta))$ in $\widetilde{G}_p$ adding the artificial source and sink and the arcs with infinite arc capacity and zero cost, connecting s' with true sources and t' with true sinks according to the following rules: for all arcs, if $\widetilde{\xi}(x_i,x_j,\vartheta,\theta)<\widetilde{u}(x_i,x_j,\vartheta,\theta)$, include the corresponding arc in $G(\widetilde{\xi}^{\mu}(x_i,x_j,\vartheta,\theta))$ with the arc capacity $\widetilde{u}^{\mu}(x_i,x_j,\vartheta,\theta)=\widetilde{u}(x_i,x_j,\vartheta,\theta)-\widetilde{\xi}(x_i,x_j,\vartheta,\theta)$, and the modified cost $\widetilde{c}^{\mu}(x_i,x_j,\vartheta,\theta)=\widetilde{c}(x_i,x_j,\vartheta,\theta)$. For all arcs, if $\widetilde{\xi}(x_i,x_j,\vartheta,\theta)>\widetilde{l}(x_i,x_j,\vartheta,\theta)$, include the corresponding arc in $G(\widetilde{\xi}^{\mu}(x_i,x_j,\vartheta,\theta))$ with the arc capacity $\widetilde{u}^{\mu}(x_j,x_i,\theta,\vartheta)=\widetilde{\xi}(x_i,x_j,\vartheta,\theta)-\widetilde{l}(x_i,x_j,\vartheta,\theta)$ and the modified cost $\widetilde{c}^{\mu}(x_j,x_i,\theta,\vartheta)=-\widetilde{c}(x_i,x_j,\vartheta,\theta)$.

Step 9. Define the minimum cost path $\widetilde{P}_p^{\mu}$ according to the Bellman–Ford algorithm from s' to t' in the constructed residual network $G(\widetilde{\xi}^{\mu}(x_i,x_j,\vartheta,\theta))$.

Step 10. Pass the flow value $\widetilde{\delta}_p^{\mu}=\min[\widetilde{u}^{\mu}(x_i,x_j,\vartheta,\theta)]$, $(x_i,x_j)\in\widetilde{P}_p^{\mu}$ along the found path.

Step 11. Update the flow values in the graph $\widetilde{G}_p$: replace the flow $\widetilde{\xi}(x_j,x_i,\vartheta,\theta)$ by $\widetilde{\xi}(x_j,x_i,\vartheta,\theta)-\widetilde{\delta}_p^{\mu}$ along the corresponding arcs, going from (x_j,ϑ) to (x_i,θ) from $\widetilde{G}_p$ for arcs, connecting node-time pair (x_i^{μ},θ) with (x_j^{μ},ϑ) in $\widetilde{G}(\widetilde{\xi}^{\mu}(x_i,x_j,\vartheta,\theta))$ with nonpositive modified cost

$\widetilde{c}^{\mu}(x_i,x_j,\theta,\vartheta)\le 0$ and replace the flow $\widetilde{\xi}(x_i,x_j,\vartheta,\theta)$ by $\widetilde{\xi}(x_i,x_j,\vartheta,\theta)+\widetilde{\delta}_p^{\mu}$ along the corresponding arcs, going from (x_i,ϑ) to (x_j,θ) from $\widetilde{G}_p$ for arcs, connecting node-time pair (x_i^{μ},ϑ) with (x_j^{μ},θ) in $\widetilde{G}(\widetilde{\xi}^{\mu}(x_i,x_j,\vartheta,\theta))$ with nonnegative modified cost $\widetilde{c}^{\mu}(x_i,x_j,\vartheta,\theta)\ge 0$ and replace the flow value in $\widetilde{G}_p$: $\widetilde{\xi}(x_i,x_j,\vartheta,\theta)\to\widetilde{\xi}(x_i,x_j,\vartheta,\theta)+\widetilde{\delta}_p^{\mu}\times\widetilde{P}_p^{\mu}$.

Step 12. Reject the artificial nodes and arcs with flows, connecting them with artificial nodes and find the total flow from the set of sources to the set of sinks for all time periods not later than p**.**

(I) If the flow value $\widetilde{\xi}(x_i,x_j,\vartheta,\theta)+\widetilde{\delta}_p^{\mu}\times\widetilde{P}_p^{\mu}$ from the set of sources to the set of sinks of minimum cost $\widetilde{c}(\widetilde{\xi}(x_i,x_j,\vartheta,\theta)+\widetilde{\delta}_p^{\mu}\times P_p^{\mu})$ for p time periods is less than the given flow value $\widetilde{\rho}(p)$, then go to **step 8**.

(II) If the flow value $\widetilde{\xi}(x_i,x_j,\vartheta,\theta)+\widetilde{\delta}_p^{\mu}\times\widetilde{P}_p^{\mu}$ for p time periods from the set of sources to the set of sinks of minimum cost $\widetilde{c}(\widetilde{\xi}(x_i,x_j,\vartheta,\theta)+\widetilde{\delta}_p^{\mu}\times P_p^{\mu})$ is equal to $\widetilde{\rho}(p)$, the given flow value of minimum cost in $\widetilde{G}_p$ is found and go to **step 13**.

(III) If the flow value $\widetilde{\xi}(x_i,x_j,\vartheta,\theta)+\widetilde{\delta}_p^{\mu}\times\widetilde{P}_p^{\mu}=\widetilde{\omega}(p)$ for p time periods from the set of sources to the set of sinks of minimum cost $\widetilde{c}(\widetilde{\xi}(x_i,x_j,\vartheta,\theta)+\widetilde{\delta}_p^{\mu}\times P_p^{\mu})$ is more than $\widetilde{\rho}(p)$ and less than $\widetilde{\nu}(p)$, then the required flow in $\widetilde{G}_p$ is $\widetilde{\xi}(x_i,x_j,\vartheta,\theta)+(\widetilde{\delta}_p^{\mu}-\widetilde{\omega}(p)+\widetilde{\rho}(p))\times\widetilde{P}_p^{\mu}$ of minimum cost $\widetilde{c}(\widetilde{\xi}(x_i,x_j,\vartheta,\theta)+(\widetilde{\delta}_p^{\mu}-\widetilde{\omega}(p)+\widetilde{\rho}(p))\times\widetilde{P}_p^{\mu})$ and go **step 13**.

Step 13. Turn to the initial dynamic graph $\widetilde{G}$ from the time-expanded static graph $\widetilde{G}_p$ as follows: the given dynamic flow of minimum cost in the graph $\widetilde{G}$ for p time periods is equal to the flow, leaving the set of sources for all time periods and entering the set of sinks for all time periods not later than p. Each path, connecting the node-time pairs (s,ϑ) with $(t,\varsigma=\vartheta+\tau_{st}(\vartheta)),\zeta\in T$, with the flow $\widetilde{\xi}(s,t,\vartheta,\varsigma)$ passing along it in $\widetilde{G}_p$ of the cost $\widetilde{c}(\widetilde{\xi}(s,t,\vartheta,\varsigma))$ corresponds to the flow $\widetilde{\xi}_{st}(\vartheta)$ of the cost $\widetilde{c}(\widetilde{\xi}_{st}(\vartheta))$ in $\widetilde{G}$.

Therefore, the proposed algorithm allows finding the minimum cost flow in a fuzzy dynamic transportation network with time-varying parameters and lower and upper fuzzy flow bounds.

IV. Numerical Example

Let us consider an example, which illustrates the implementation of the algorithm. Let the transportation network, which is the part of railway network, be presented as a fuzzy directed network, obtained from GIS "Object Land" [14], as shown in Fig. 1.

The node x_1 is the source, the node x_6 is the sink. Fuzzy flow bounds, arc costs and crisp parameters of time, which depend on the flow departure, are presented in Tables I, II, III.

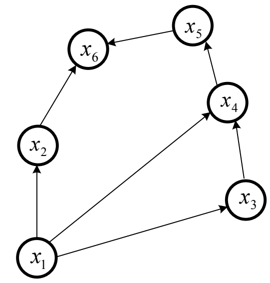

Fig. 1. Initial dynamic graph $\widetilde{G}$

TABLE I
Flow Bounds Dependent On The Flow Departure

$\widetilde{u}_{ij}/\widetilde{l}_{ij}$ \ θ	0	1	2	3
(x_1,x_2)	(10,1.5,2)	(10,1.5,2) (18,3,3)	(8,1,1)	(10,1.5,2)
(x_1,x_3)	(10,1.5,2)	(15,3,2)	(18,3,3)	(18,3,3)
(x_1,x_4)	(18,3,3) (20,3,4)	(20,3,4)	(25,4,5)	(18,3,3)
(x_2,x_6)	(30,5,6)	(25,4,5)	(40,7,7)	(30,5,6)
(x_3,x_4)	(25,4,5)	(18,3,3)	(20,3,4)	(20,3,4)
(x_4,x_5)	(30,5,6)	(30,5,6)	(25,4,5)	(25,4,5)
(x_5,x_6)	(30,5,6)	(30,5,6)	(45,8,8)	(25,4,5)

TABLE II
Transmission Costs Dependent on the Flow Departure

$\widetilde{c}_{ij}$ \ θ	0	1	2	3
(x_1,x_2)	(30,7,6)	(60,10,9)	(60,10,9)	(30,7,6)
(x_1,x_3)	(75,11,12)	(50,9,8)	(18,3,3)	(18,3,3)
(x_1,x_4)	(70,10,10)	(30,7,6)	(20,3,4)	(18,3,3)
(x_2,x_6)	(30,7,6)	(25,4,5)	(50,9,8)	(30,7,6)
(x_3,x_4)	(25,4,5)	(30,7,6)	(80,15,15)	(20,3,4)
(x_4,x_5)	(30,7,6)	(100,20,17)	(80,15,15)	(25,4,5)
(x_5,x_6)	(30,7,6)	(30,7,6)	(80,15,15)	(25,4,5)

TABLE III
Transmission Times Dependent on the Flow Departure

τ_{ij} \ θ	0	1	2	3
(x_1,x_2)	5	1	3	2
(x_1,x_3)	1	3	2	1
(x_1,x_4)	1	3	3	3
(x_2,x_6)	4	4	1	2
(x_3,x_4)	5	1	2	3
(x_4,x_5)	4	1	1	1
(x_5,x_6)	4	4	1	1

Add the artificial nodes and arcs, connecting them with other nodes to the constructed graph in Fig. 2 and turn to the graph $\widetilde{G}_p^*$ without lower flow bounds, which is shown in Fig. 3.

Connectors, which have the same shape (for example, ▲), connect the corresponding pair of nodes in Fig. 3. Therefore, each node x_6 for all time periods is connected with each node x_1 for all time periods. It is represented by the connectors of the same shape. All arcs, going from x_6 for all time periods to the nodes x_1 for all time periods have infinite upper flow bounds, zero lower flow bounds and zero costs.

Applying the steps of the algorithm, we find the paths of minimum cost. Find the fist path of minimum cost from s^* to t^* according to the Bellman–Ford algorithm in the residual network, which initially corresponds to $\widetilde{G}_p^*$. We get two identical paths of the minimum cost: $s^*,(x_2,2),(x_6,3),(x_1,1),t^*$ and $s^*,(x_2,2),(x_6,3),(x_1,0),t^*$ of the cost (50,9,8) conventional units.

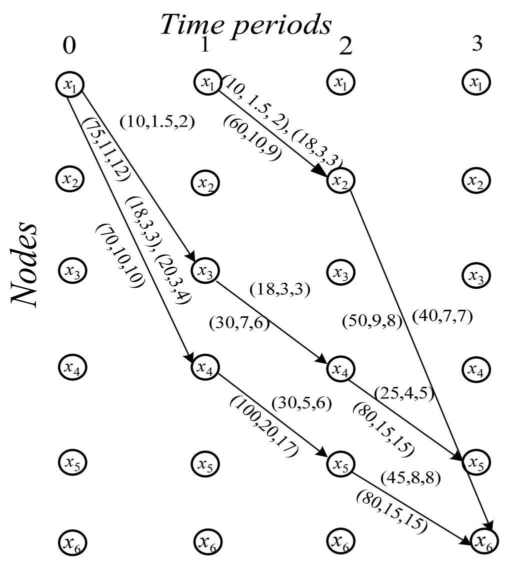

Fig. 2. Expanded graph $\widetilde{G}_p$ of the initial dynamic graph $\widetilde{G}$

Let us choose the first path and push min from $[(10,1.5,2),(40,7,7),\infty,(10,1.5,2)]$, i.e., (10,1.5,2) flow units along it, i.e., the flow $\widetilde{0}$ goes to $(10,1.5,2)\times\widetilde{P}_1^{*\mu}$. Find the second path of minimum cost $\widetilde{P}_2^{*\mu}$ according to the Bellman–Ford algorithm in the residual network $\widetilde{G}_p^{*\mu}$: $\widetilde{P}_2^{*\mu}=s^*,(x_4,1),(x_5,2),(x_6,3),(x_1,0),t^*$. Push min from $[(18,3,3),(30,5,6),(45,8,8),\infty,(18,3,3)]$, i.e., (18,3,3) flow units along the path $\widetilde{P}_2^{*\mu}=s^*,(x_4,1),(x_5,2),(x_6,3),(x_1,0),t^*$, i.e., flow $(10,1.5,2)\times\widetilde{P}_1^{*\mu}$ goes to $(10,1.5,2)\times\widetilde{P}_1^{*\mu}+(18,3,3)\times\widetilde{P}_2^{*\mu}$. The flow value $(10,1.5,2)\times\widetilde{P}_1^{*\mu}+(18,3,3)\times\widetilde{P}_2^{*\mu}$ is equal to $\sum_{\widetilde{l}(x_i,x_j,\vartheta,\theta)\neq\widetilde{0}}\widetilde{l}(x_i,x_j,\vartheta,\theta)$, so the maximum flow is found in the expanded graph $\widetilde{G}_p^*$ with introduced artificial arcs and nodes. This flow is equal to the sum of lower flow bounds: $(10,1.5,2)+(18,3,3)$, i.e., (28,4,5) units. Therefore, there is the feasible flow in $\widetilde{G}_p$ and it is equal to the total flow, passing along the reverse arcs, connecting the nodes $(x_5,\forall\theta\in T)$ with $(x_1,\forall\theta\in T)$ for all time periods, i.e., (28,4,5) units.

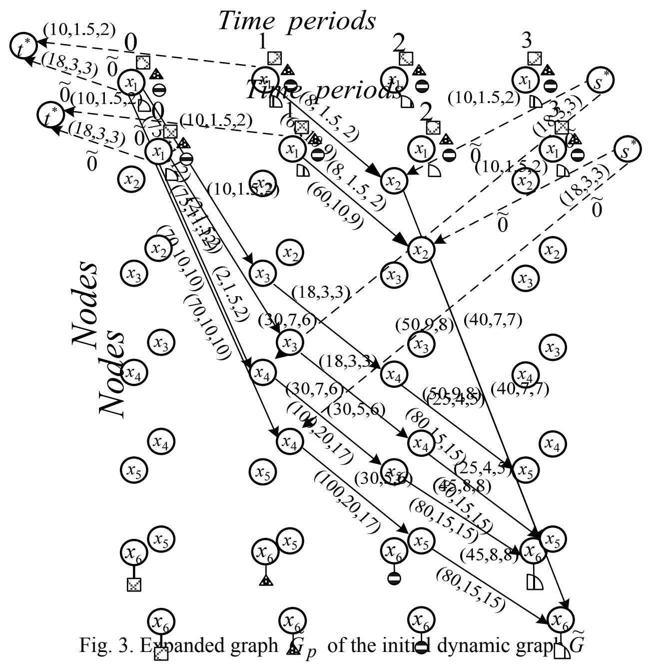

Fig. 3. Expanded graph $\widetilde{G}_p$ of the initial dynamic graph $\widetilde{G}$

This flow value is less than the given flow rate (30,5,6) units; therefore, we turn to determining the given flow cost in the initial expanded graph. Construct the network with flow $\widetilde{G}_p(\widetilde{\xi})$, deleting artificial nodes and arcs and taking into account that the feasible flow vector $\widetilde{\xi}=(\widetilde{\xi}(x_i,x_j,\vartheta,\theta))$ of the value $\widetilde{\sigma}$ is defined as $\widetilde{\xi}(x_i,x_j,\vartheta,\theta)=\widetilde{\xi}^*(x_i,x_j,\vartheta,\theta)+\widetilde{l}(x_i,x_j,\vartheta,\theta)$, where $\widetilde{\xi}^*(x_i,x_j,\vartheta,\theta)$ – the flows, passing along the arcs of the graph $\widetilde{G}_p^*$ after deleting the artificial nodes and connected arcs. Construct a network with the feasible flow, as shown in Fig. 4.

Introduce the artificial source and sink, connecting them with the true sources and sinks by the arcs with infinite arc capacities and construct the residual network for the graph in Fig. 4, as shown in Fig. 5. Find a path of minimum cost $\widetilde{P}_1^{\mu}$ according to the Bellman–Ford algorithm in the residual network $\widetilde{G}(\widetilde{\xi}^{\mu}(x_i,x_j,\vartheta,\theta))$: $\widetilde{P}_1^{\mu}=s',(x_1,1),(x_2,2),(x_6,3),t'$. Pass min from $[\infty,(8,1.5,2),(30,5,6),\infty]$, i.e., (8,1.5,2) flow units along this path, the feasible flow turns to $\widetilde{\xi}(x_i,x_j,\vartheta,\theta)+(8,1.5,2)\times\widetilde{P}_1^{\mu}$ from $\widetilde{\xi}(x_i,x_j,\vartheta,\theta)$, therefore, the flow value $(28,4,5)+(8,1.5,2)\times\widetilde{P}_1^{\mu}$ exceeds the given flow (30,5,6), thus, the given flow value can be found as

$(28{,}4{,}5)+((8{,}1.5{,}2)-(36{,}6{,}7)+(30{,}5{,}6))\times(s',(x_1,1),$
$(x_2,2),(x_6,3),\ t')=((28{,}4{,}5)+(2{,}1.5{,}2))\times(s',(x_1,1),$
$(x_2,2),(x_6,3),\ t').$

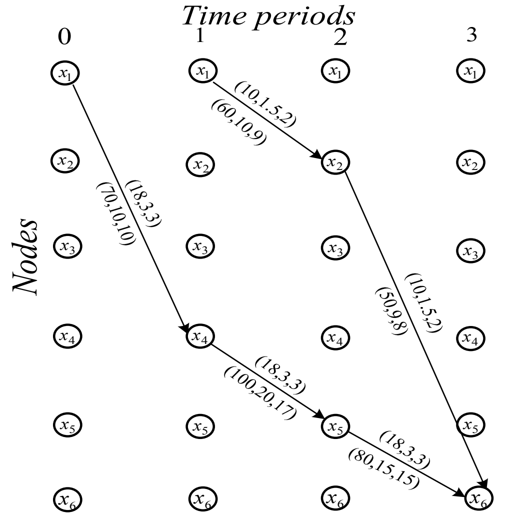

Fig. 4. Graph with the feasible flow $\tilde{G}(\tilde{\xi})$

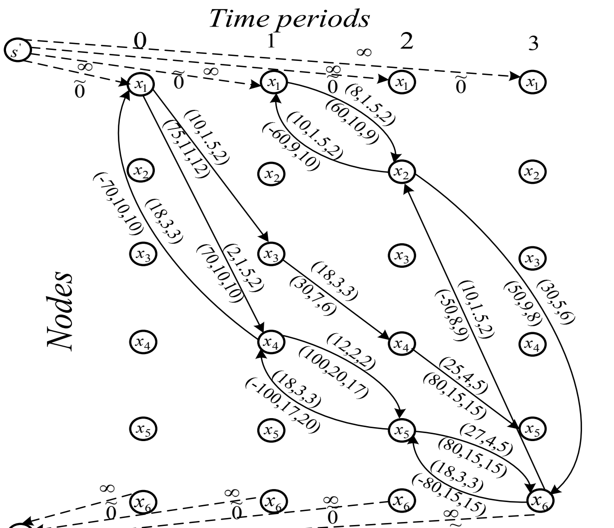

Fig. 5. The residual network $\tilde{G}(\tilde{\xi}^{\mu}(x_i,x_j,\vartheta,\theta))$ for the graph $\tilde{G}(\tilde{\xi})$

Therefore, the graph with a feasible glow is shown in Fig. 6. The found feasible flow (30,5,6) has minimum cost $(18{,}3{,}3)\times((70{,}10{,}10)+(100{,}20{,}17)+(80{,}15{,}15))+$ $(12{,}1.5{,}2)\times((60{,}10{,}9)+(50{,}9{,}8))=(5820{,}20{,}17)$ conventional units.

Turning to dynamic graph $\tilde{G}$ from expanded static graph $\tilde{G}_p$, we come to a conclusion that the given flow value for 3 time periods is equal to the flow, leaving from the "node-time" pairs $(x_1,0)$ and $(x_1,1)$ and entering the "node-time" pair $(x_6,3)$, i.e., (30,5,6) flow units, which are defined by a path $x_1 \to x_4 \to x_5 \to x_6$, which departs at $\theta=0$ and arrives at the sink at $\theta=3$ and by a path $x_1 \to x_2 \to x_6$, which departs at $\theta=1$ and arrives at the sink $\theta=3$.

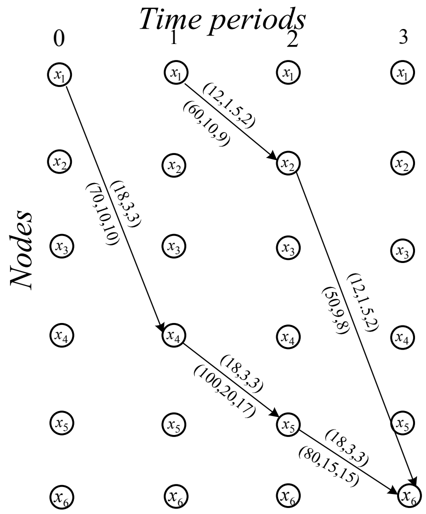

Fig. 6. Graph $\tilde{G}_p$ with the feasible flow $(30,5,6)$

V. Conclusion

This article examines the task of minimum cost flow finding in a dynamic graph. The distinguishing feature of the problem lies in the fuzzy nature of network parameters. The relevance of this task is that the time factor and the tendency of the flow bounds, costs and transit times change over time, when finding the minimum cost flow for the given number of time periods. The necessity of introducing the lower bounds is taken into account due to the complex nature of the network.

Acknowledgments

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Alexander Bozhenyuk, Professor of Informatics at the Faculty of Information Security, Southern Federal University, Russia. He holds a degree of Doctor of Technical Sciences in Theoretical Foundations of Informatics.
His research interests include fuzzy sets, fuzzy logic, fuzzy graphs and fuzzy hypergraphs. He has more than 180 publications in the field of Computer Science. Contact information: 44, Nekrasovsky Street, Taganrog, 347900, Russia; phone: +78634371743; e-mail: avb002@yandex.ru.

Evgeniya Gerasimenko is a postgraduate student at the Faculty of Information Security, Southern Federal University, Russia. Her research interests include fuzzy flows in networks, dynamic flows. She has more than 15 publications in the field of Computer Science.
Contact information: 44, Nekrasovsky Street, Taganrog, 347900, Russia; phone: +78634371743; e-mail: e.rogushina@gmail.com.

Aleksandrs Boženuks, Jevgeņija Ģerasimenko. Minimālās izmaksas monitoringa algoritms izplūdušajos dinamiskajos tīklos
Šajā rakstā tiek aplūkota plūsmas minimālās izmaksas izplūdušā dinamiskā transporta tīklā problēma, ņemot vērā plūsmu zemākās robežas, kuras definētas kā grafa loki. Literatūrā plūsmas ir definētas kā dinamiskie grafi, bet tie neattēlo parametru izplūdušo raksturu transporta tīklā, caurlaides spējas atkarību, pārvadājumu izmaksas un laika parametrus no plūsmas izsūtīšanas sākuma. Aplūkojamās problēmas īpatnības ir saistītas ar to, ka transporta tīkla parametri, tādi kā augšējās plūsmas robežas, apakšējās plūsmas robežas un izmaksas, tiek definēti izplūdušā veidā, jo uz tiem iedarbojas apkārtējās vides faktori, mērījumu kļūdas un benzīna cenu izmaiņas. Problēma ietver arī transporta tīkla parametru atkarību no plūsmas izsūtīšanas sākuma laika, kas norāda uz aplūkojamā transporta tīkla dinamiskumu, pretstatā klasiskajā literatūrā aplūkotajam stacionāri-dinamiskām plūsmām. Lai atrisinātu rakstā aplūkoto problēmu, tika izstrādāts algoritms, balstīts uz „laikā izstieptu" grafu, kas nodrošina korektu pāreju no izplūduša dinamiska grafa uz statisku izejas grafa variantu. Pēc pārejas uz statisku izejas grafa variantu tiek meklētas minimālo izmaksu saites, pa kurām tiek novadīta plūsma, kamēr tiek sasniegts nepieciešamais plūsmas līmenis. Rakstā piedāvātais algoritms var tikt izmantots reālā ceļu tīklā, lai izpildītu uzdevumus, kas risina pārvadājumu maršrutu minimālo izmaksu atrašanu, ņemot vērā laika diapazonu, kurā plūsmai ir jābūt pārvadītai. Algoritma darbības rezultātā tiek iegūta plūsmas minimālās izmaksas nepieciešamā vērtība, kas tiek definēta izplūdušā veidā un nodrošina pāreju uz dinamisku grafu.

Александр Боженюк, Евгения Герасименко. Алгоритм мониторинга минимальной стоимости в нечётких динамических сетях
Данная статья рассматривает задачу нахождения потока минимальной стоимости в нечеткой динамической транспортной сети с учетом нижних границ потоков, заданных на дугах графа. В литературе по потокам встречаются постановки задач на динамических графах, но они не учитывают нечеткий характер параметров транспортной сети, а также зависимость пропускных способностей, стоимостей перевозок и параметров времени от момента отправления потока. Особенность постановки задачи в том, что параметры транспортной сети, такие как нижние и верхние границы потока и стоимости задаются в нечетком виде, поскольку на эти параметры влияют факторы окружающей среды, погрешности в измерениях, колебания в ценах на бензин. Также данная постановка задачи предполагает зависимость параметров транспортной сети от времени отправления потока, что позволяет считать рассматриваемую транспортную сеть истинно динамической в отличие от «стационарно-динамических», рассматриваемых в классической литературе по потокам. Для решения поставленной проблемы был разработан алгоритм на основе «растянутого во времени» графа, позволяющий корректно перейти от нечеткого динамического графа к нечеткому статическому варианту исходного графа. После перехода к статическому варианту исходного графа, осуществляется поиск цепей минимальной стоимости, по которым передается поток, до тех пор, пока не будет получено требуемое значение потока. Алгоритм, предлагаемый в данной статье, может быть использован на реальных сетях дорог для решения задач нахождения маршрутов перевозки минимальной стоимости, учитывающих временной диапазон, в течение которого поток должен быть перевезен. В результате работы алгоритма мы получаем требуемое значение потока найденной минимальной стоимости, заданное в нечетком виде, и осуществляем обратный переход к динамическому графу.