Flexible Neo-fuzzy Neuron and Neuro-fuzzy Network for Monitoring Time Series Properties

Yevgeniy Bodyanskiy, Iryna Pliss, Olena Vynokurova

Abstract – In the paper, a new flexible modification of neo-fuzzy neuron, neuro-fuzzy network based on these neurons and adaptive learning algorithms for the tuning of their all parameters are proposed. The algorithms are of interest because they ensure the on-line tuning of not only the synaptic weights and membership function parameters but also forms of these functions that provide improving approximation properties and allow avoiding the occurrence of “gaps” in the space of inputs.

Keywords – Flexible activation-membership function, flexible neo-fuzzy neuron, forecasting, identification learning algorithm

I. INTRODUCTION

Artificial neural networks (ANN), fuzzy inference systems (FIS) and wavelet systems (WS) have been widely used in recent years to solve a wide range of problems, such as Dynamic Data Mining [1], [2] and processing of nonlinear non-stationary signals of different nature under a priori and current uncertainty.

Hybrid wavelet-neuro-fuzzy systems [3], [4], [5], [6] emerged as the synergism of these three directions in computational intelligence. The wavelet-neuro-fuzzy systems possess the learning capabilities similar to those of neural networks, provide the interpretability and transparency of results inherent to the fuzzy approach and similarly effective wavelet systems for non-stationary signal processing with local features.

The main disadvantages of wavelet-neuro-fuzzy systems especially when using in an on-line mode are related to the slow convergence of the conventional gradient-based learning procedures and computational complexity of second-order procedures when using in sequential adaptive variants. Furthermore, significant problems may arise in the processing of non-stationary signals, since the second-order procedures, for example, exponentially weighted recurrent least squares method can be numerically instable.

II. NEO-FUZZY NEURON AND ITS ARCHITECTURE

To overcome these difficulties, a hybrid neuro-fuzzy system called “neo-fuzzy-neuron” (Fig. 1) was proposed in [7], [8], [9]. As it can be seen, the architecture of neo-fuzzy neuron is quite close to the conventional n -inputs artificial neuron; however, instead of usual synaptic weights \( w_{ji} \), it contains the so-called nonlinear synapses \( N\lambda_j, j = 1,2,K,n \).

When an input vector signal \( x(k) = (x_1(k), x_2(k), K, x_n(k))^T \) (here \( k = 1,2,K \) is a current discrete time) is fed to the input of the neo-fuzzy neuron, its output is defined by both the membership functions \( m_{ji}(x_i(k)) \) (its authors have used conventional triangular functions) and the tunable synaptic weights \( w_{ji}(k) \):

\[
\hat{y}(k) = \sum_{i=1}^{n} f_i(x_i(k)) = \sum_{i=1}^{n} \hat{a}_{ji} k = 1,2,K \quad \text{and} \quad w_{ji}(k) - 1 m_{ji}(x_i(k)). \quad (1)
\]

Using the learning criterion in the form of standard quadratic error function

\[
E(k) = \frac{1}{2} (y(k) - \hat{y}(k))^2 = \frac{1}{2} e^2(k) \quad (2)
\]

minimized via the usual gradient descent procedure, it is easy to write weights update algorithm in the form

\[
w_{ji}(k + 1) = w_{ji}(k) + he(k + 1)m_{ji}(x_i(k + 1)), \quad (3)
\]
where $y(k)$ is the target value of the output (reference signal), $h$ is the learning rate parameter, which determines the rate of the convergence and as a rule is chosen empirically.

The authors of neo-fuzzy neuron note [9] that among its most important advantages there are the rate of learning, high approximation properties, computational simplicity, the possibility of finding the global minimum of the learning criterion in real time.

At the same time, the efficiency of this system is reduced when processing non-stationary signals disturbed by noises of unknown nature. To increase the learning rate in [10], an adaptive optimal learning algorithm was proposed in the form

$$w_p(k + 1) = w_p(k) + \frac{e(k + 1) m(x(k + 1))}{\|m(x(k + 1))\|^2}$$  \hspace{1cm} (4)

which is a variety of the known optimal Kaczmarz-Widrow-Hoff procedure. Here $w(k) = (w_{i1}(k), w_{i2}(k), K, w_{in}(k), w_{i0}(k), K, w_{in}(k))^T$, $m(k) = (m_{i1}(x_i(k)), K, m_{i2}(x_i(k)), K, m_{in}(x_i(k)), K, m_{in}(x_i(k)))^T$. $m$, $m_i$, $w_i$ are $i = 1, K, n; j = 1, K, h$.

To provide both tracking (in non-stationary situation) and filtering (when stochastic disturbances corrupt the processed signal) properties for the neo-fuzzy neuron, a modified adaptive procedure was used

$$\begin{align*}
\dot{w}(k + 1) &= w(k) + r^{−1}(k + 1)e(k + 1)m(x(k + 1)), \\
\dot{r}(k + 1) &= b r(k + 1) + \|m(x(k + 1))\|, \\
0 \leq b \leq 1
\end{align*}$$  \hspace{1cm} (5)


Further modifications of neo-fuzzy neuron were connected to improve its approximation properties. Thus, in [10] instead of the triangular membership functions the second-degree polynomials were proposed, and in [12] the fourth-degree polynomials were recommended. In [13], [14] cubic and B-splines functions were proposed to be used, and in [4], [5], [15], [16] different types of odd wavelets were suggested.

III. FLEXIBLE NEO-FUZZY NEURON WITH TUNABLE FORM OF MEMBERSHIP FUNCTIONS

Let us introduce an activation-membership function that is described by the expression

$$m_p(x_p(k)) = \left(1 - a_p(k)t_p(k)\right)^{s_p(k)} \exp\left[\frac{f_p(k)^{s_p(k)}}{s_p(k)}\right]$$  \hspace{1cm} (6)

where $t_p(k) = (x(k) - c_p(k))s_p(k); c_p(k)$ is the centre of activation-membership function; $s_p(k)$ is the width of activation-membership function; $a_p(k)$ is the shape parameter of activation-membership function; $s_p(k)$ is the flexible parameter of activation-membership function.

It is obvious that when $a = 0, s = 2, s = 1$ $m_p(x_p(k))$ is the Gaussian function (Fig. 2a), when $a = 1, s = 2, s = 1$ – the wavelet Mexican Hat (Fig. 2b), $a = 0, s = 100, s = 3$ – the trapezoidal-like function (Fig. 2d), $a = 0, s = 1, s = 8$ – the triangular-like function (Fig. 2e) etc. (Fig. 2c, Fig. 2f).

Figure 2 shows the forms of such a function for different parameter values.


\[ J^w_p(k) = m_p(x(k)), \]

\[ J^w_p(k) = w_p(k) + r^w_p(k) \left( r^w_p(k) + 1 \right)^{-1}, \]

\[ c_p(k + 1) = c_p(k) + e(k + 1) J^w_p(k + 1) \left( r^w_p(k + 1) \right)^{-1}, \]

\[ r^w_p(k + 1) = b r^w_p(k) + P J^w_p(k + 1) \mathbf{P}^+, \]

\[ s_p(k + 1) = s_p(k) + e(k + 1) J^w_p(k + 1) \left( r^w_p(k + 1) \right)^{-1}, \]

\[ r^w_p(k + 1) = b r^w_p(k) + P J^w_p(k + 1) \mathbf{P}^+, \]

\[ a_p(k + 1) = a_p(k) + e(k + 1) J^w_p(k + 1) \left( r^w_p(k + 1) \right)^{-1}, \]

\[ r^w_p(k + 1) = b r^w_p(k) + P J^w_p(k + 1) \mathbf{P}^+, \]

\[ s_p(k + 1) = s_p(k) + e(k + 1) J^w_p(k + 1) \left( r^w_p(k + 1) \right)^{-1}, \]

\[ r^w_p(k + 1) = b r^w_p(k) + P J^w_p(k + 1) \mathbf{P}^+, \]

\[ \text{IV. FLEXIBLE NEURO-FUZZY NETWORK WITH TUNABLE FORM OF MEMBERSHIP FUNCTIONS} \]

The neuro-fuzzy network proposed by Wang and Mendel [17], which possesses universal approximating capabilities, has a sufficiently close architecture to the neo-fuzzy neuron. Figure 3 shows the architecture of such a network without a normalization layer.

This neuro-fuzzy network implements scatter input space partition [18] and consists of \( h \) tunable weights (in \( n \) time less than that for the neo-fuzzy-neuron). The normalization layer is needed for that space partition to avoid the emergence of “gaps” in the input space.

Of course, the emergence of “gaps” can be avoided using the grid input space partition, but at the same time the number of tuning synaptic weights sharply increases and achieves value \( h^2 \).

When optimizing procedure (7) on its rate of convergence and introducing tracking and filtering properties we get a final expression:

\[ \dot{y}(k) = \sum_{j=1}^{h} w_j(k-1) y_j(x(k)) = \sum_{j=1}^{h} \hat{w}_j(k-1) \hat{m}_j(x(k)) \]

and for its tuning the common learning criterion (2) can be used.

If we introduce derivatives of this criterion on all tuning parameters of network, we can rewrite (8) in the form

\[ \dot{y}(k) = \sum_{j=1}^{h} w_j(k-1) y_j(x(k)) = \sum_{j=1}^{h} \hat{w}_j(k-1) \hat{m}_j(x(k)) \]
\[ J^*_\mu(k) = y_\mu(x(k)) = \hat{\Omega} \left[ \sum_{i=1}^{m_\mu} \begin{pmatrix} m_\mu \cdot (x_i(k)) \\ \end{pmatrix} \right] + \begin{pmatrix} m_\mu \cdot (x_i(k)) \\ \end{pmatrix} \cdot \frac{1}{s_{\mu}^2(k)} + \begin{pmatrix} m_\mu \cdot (x_i(k)) \\ \end{pmatrix} \cdot \frac{1}{s_{\mu}^2(k)} \]

\[ J^*_\mu(k) = w_\mu \left( \begin{pmatrix} m_\mu \cdot (x_i(k)) \\ \end{pmatrix} \right) \cdot \frac{1}{s_{\mu}^2(k)} + \begin{pmatrix} m_\mu \cdot (x_i(k)) \\ \end{pmatrix} \cdot \frac{1}{s_{\mu}^2(k)} \cdot \ln \left( \begin{pmatrix} m_\mu \cdot (x_i(k)) \\ \end{pmatrix} \cdot \frac{1}{s_{\mu}^2(k)} \right) \]

\[ \hat{x}(t) = \frac{0.2x(t-\tau)}{1 + x^{100}(t-\tau)} - 0.1x(t) \] (12)

Values of the time series (12) were computed in each integer-valued point by the Runge-Kutta fourth-order method. Time step was taken equal to 0.1, initial values of the network parameters were taken as \( x(0) = 1.2 \) and \( t = 17 \) to obtain the values in the interval \( t = 0 \) to 5000.

Values \( x(k - 18), x(k - 12), x(k - 6), x(k) \) were taken as pre-history to predict value \( x(k + 6) \). Parameters of learning algorithm were taken as \( b = 0.99 \), \( r^* = r^* = r^* = r^* = 10000 \).

After 50000 steps, the learning process was stopped and next 500 samples were used for prediction. Initial values of the synaptic weights were obtained randomly in the interval \([-0.1, 0.1]\).

As a quality criterion, we have used two types of error: root mean square error (RMSE) and absolute percentage error (APE).

Figure 4 shows the prediction results of chaotic time series. As it can be seen, the real signal (dot line) and forecast signal (solid line) are almost indistinguishable.

Table I shows the comparative analysis of chaotic time series prediction based on different approaches.

<table>
<thead>
<tr>
<th></th>
<th>Neural network / Learning algorithm</th>
<th>RMSE</th>
<th>APE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flexible neo-fuzzy neuron / Proposed learning algorithm</td>
<td>0.0091</td>
<td>2.8%</td>
<td></td>
</tr>
<tr>
<td>Flexible neuro-fuzzy network / Proposed learning algorithm</td>
<td>0.0101</td>
<td>3.1%</td>
<td></td>
</tr>
<tr>
<td>Flexible neo-fuzzy neuron / Gradient learning algorithm</td>
<td>0.0125</td>
<td>3.92%</td>
<td></td>
</tr>
<tr>
<td>Radial basis function network / Learning algorithm (all-parameters)</td>
<td>0.0231</td>
<td>4.99%</td>
<td></td>
</tr>
<tr>
<td>Radial basis function network / Learning algorithm (only synaptic weights)</td>
<td>0.0595</td>
<td>5.89%</td>
<td></td>
</tr>
<tr>
<td>Multilayer feed forward neural network / Gradient learning algorithm</td>
<td>0.2132</td>
<td>11.35%</td>
<td></td>
</tr>
<tr>
<td>ANFIS/ Gradient learning algorithm</td>
<td>0.0198</td>
<td>4.12%</td>
<td></td>
</tr>
</tbody>
</table>

Thus, as it can be seen from experimental results, the proposed approach provides the best quality of prediction in comparison with conventional approaches due to the tuning of all parameters of a flexible neo-fuzzy network.
VI. CONCLUSION

Flexible neo-fuzzy neuron and neuro-fuzzy network with membership functions of variable tunable forms and adaptive learning algorithms with tracking and filtering properties have been proposed. The learning algorithms are simple in implementation and provide the high quality of signal processing in an on-line mode. Tuning of activation-membership function form improves the accuracy of modelling of nonlinear non-stationary processes. This has been shown in the experiments of different time series forecasting. The proposed approach may be effectively used in many Dynamic Data Mining tasks, namely in the monitoring of time series property tasks.

REFERENCES

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Evgeniy Bodianskiy, Irina Pliss, Olena Vynokurova. Гибкий нео-фаззи нейрон и нео-фаззи сеть для мониторинга свойств временных рядов

В настоящее время искусственные нейронные сети, фаззий-системы и интеллектуальные системы получили широкое распространение для решения задач динамического интеллектуального анализа данных и обработки нестационарных нелинейных сигналов производной информации в условиях априорной и текущей неопределенности. Гибризизация преимуществ каждой из систем позволяет решать сложные задачи динамического интеллектуального анализа данных на новом качественном уровне. Также большинство предложенных алгоритмов обучения обладают низкой скоростью сходимости и не позволяют решать задачи в on-line режиме, что особенно важно в задачах медицинского мониторинга, экономического прогнозирования и др. В статье предложена новая гибкая модификация нео-фаззи нейрон и нео-фаззи сети, а также алгоритмы обучения всех параметров. В архитектуре гибкого нео-фаззи нейрона введён новый тип функций активации-принадлежности, которые имеют четыре настраиваемых параметра: центр, ширина, форма и гибкость. Предложенный алгоритм обучения обеспечивает оптимальную настройку в on-line режиме не только синаптических весов, но и всех параметров гибких функций активации-принадлежности, что обеспечивает улучшенные аппроксимирующие свойства алгоритма, а также позволяет избежать появление “дырок” в разбиении входного пространства. Предложенный гибкий нео-фаззи нейрон и модифицированный неофаззи сеть могут быть использованы как самостоятельные нейронные сети, а также как строительные элементы эволюционирующих неофаззи систем и ансамблей нейронных сетей. Проведённое имитационное моделирование подтверждает преимущества предлагаемого подхода. Предлагаемые системы позволяют решать задачи прогнозирования, мониторинга, идентификации, а также эмуляции нестационарных хаотических временных рядов технической, медицинской, финансовой, экологической, природной.