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Variational Maximum Likelihood Method and Its Application to Aerospace Navigation

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Abstract **– Variational approach to statistical evaluation of a maximum likelihood state in a nonlinear dynamic system is proposed. Mathematical justification of the approach and comparison with the direct methods showed its advantages concerning the obtained estimates, accuracy, and computational efficiency. Numerical examples of autonomous orbit determination according to navigational data are considered.**

Keywords **– Boundary conditions, boundary problem, maximum likelihood method, navigation, quality analysis, statistical estimation**

I. INTRODUCTION

Routine methods of orbit determination use the direct conditions of the maximum likelihood method (MLM) and implicate the joint processing of measuring data for a complete sample via the computation of first-order ballistic derivatives [1]-[5], [10]-[12]. Appropriate improvements are investigated thoroughly enough. The use of second-order derivatives improves the convergence and conditionality, though this essentially increases intensity of computations [13], [14]. Variatonal approach to statistical estimation is an effective alternative [6]-[9]. The main point of the investigation is the substantiation and use of variational necessary conditions of optimality. The development of variational technology will allow considering the multi-point problems of optimal control and the navigational estimation problems within the common context and use similar methods.

The aim of the current research is the justification of a variational variant of MLM. New mathematical basics of navigational algorithms are also considered in the paper.

II. PROBLEM DEFINITION

Task 1. Let object dynamics be described via the following vectorial differential equation:

$$\dot{\bar{x}} = \bar{\varphi}(\bar{x}, \bar{c}, t), \quad \bar{x}(t_0) = \bar{x}_0, \quad t \in [t_0, T]. \tag{1}$$

The measurements involve the following m-dimensional vector:

$$\bar{\psi}(t) = \bar{\psi}[\bar{x}(t)].$$

Let $\bar{y}(t_i) = \bar{y}_i$ be a measured value of the vector $\bar{\psi}$ at the time point t_i. Then we get the following measuring model:

$$\bar{y}(t_i) = \bar{\psi}[\bar{x}(t_i)] + \bar{\delta}_i\,, \; i = 1(1)N \,;.\, t_i \in [t_0, T]. \tag{2}$$

Here $\bar{\delta}_i$ is an m-dimensional vector of random measuring errors following a multivariate continuously differentiable stochastic distribution $f(\bar{\delta}_i, \bar{\alpha}_i)$ with parameters $\bar{\alpha}_i$. This distribution needs not be normal in a general case.

Our aim is to determine estimates of the vectors $\bar{x}_0$ and $\bar{c}$ providing minimum for the functional:

$$I = \sum_{i=1}^{N} \rho_i \left\{ \bar{y}(t_i), \bar{\psi}[\bar{x}(t_i)], \bar{\alpha}_{\mathrm{i}} \right\}, \tag{3}$$

where

$$\rho_i = \ln f_i \left\{ \bar{y}(t_i) - \bar{\psi}[\bar{x}(t_i), \bar{\alpha}_i] \right\}, \; i = 1(1)N\,. \tag{4}$$

We assume that the functions $\bar{\varphi}(\bar{x}, \bar{c}, t)$ and $\bar{\psi}[\bar{x}(t_i)]$ are bounded and continuously differentiable for all arguments over the whole domain.

We see that functional (3) is nothing else than a logarithmic likelihood function.

We assume that the observability conditions [3] hold.

III. VARIATIONAL CONDITIONS OF OPTIMALITY

To define functional (3) in an integral form we introduce the function

$$\rho\{\bar{y}(t), \bar{\psi}[\bar{z}(t)], \bar{\alpha}(t)\} == \ln f \left\{ \bar{y}(t) - \bar{\psi}[\bar{x}(t)], \bar{\alpha}(t) \right\}, \tag{5}$$

where $\bar{y}(t)$ and $\bar{\alpha}(t)$ are arbitrary continuously differentiable vectorial functions (for example, Lagrange polynomials) taking time points t_i to $\bar{y}_i$ and $\bar{\alpha}_i$, respectively. Now we get the following formula for functional (3)

$$I = \int_{t_0}^{T} \rho \left\{ \bar{y}(t), \bar{\psi}[\bar{x}(t)] \right\} \sum_{i=1}^{N} \delta(t - t_i) dt\,, \tag{6}$$

here $\delta(t - t_i)$ is an impulse δ-function.

The additional vector $\bar{x}_1(t) = \bar{c}$ and the system

$$\dot{\bar{x}}_1(t) = \bar{\varphi}_{x_1}(\bar{x}_1, t) \equiv 0\,; \qquad \bar{x}_1(t_0) = \bar{c} \tag{7}$$

allow defining the state space. Moreover, task 2 for the extended k-dimensional ($k = n + l$) state vector $\bar{z}(t) = [\bar{x}(t), \bar{x}_1(t)]^T$ substitutes for task 1.

Task 2. Under the conditions of:

$$\begin{gathered} \dot{\bar{z}} = \bar{\varphi}_z(\bar{z}, t)\,; \qquad \bar{\varphi}_z = [\bar{\varphi}, \bar{\varphi}_{x_1}]^T\,; \\ \bar{y}(t_i) = \bar{\psi}_i[\bar{z}(t_i)] + \bar{\delta}_i\,; \\ I = \int_{t_0}^{T} \rho \left\{ \bar{y}(t), \bar{\psi}[\bar{z}(t)] \right\} \sum_{i=1}^{N} \delta(t - t_i) dt \end{gathered} \tag{8}$$

an optimal estimate $\bar{z}_0$ is to be evaluated.

The following necessary conditions of optimality for the estimate $\bar{z}_0$ were obtained via the well-known procedures of variational calculus.

Theorem. Optimal estimate $\bar{z}_0$ for task 2 and the corresponding optimal trajectory $\bar{z}(\bar{z}_0, t)$ establish a solution for a two-point boundary problem defined via the following canonical system

$$\dot{\bar{z}} = \frac{\partial H}{\partial \bar{\lambda}_z}; \qquad \dot{\bar{\lambda}}_z = -\frac{\partial H}{\partial \bar{z}}, \tag{9}$$

under boundary conditions

$$\bar{\lambda}_z(t_0) = 0; \quad \bar{\lambda}_z(T) = 0;$$

$$\bar{\lambda}_z(t_i^+) = \bar{\lambda}_z(t_i^-) + \frac{\partial \rho}{\partial \bar{z}}\{\bar{y}_i, \bar{\psi}[\bar{z}(t_i)]\};$$

where

$$H = \bar{\lambda}_z^T \bar{\varphi}_z;$$

$\bar{\lambda}_z$ is a k -dimensional vectorial function.

Hence, the task can be regarded as a two-point boundary problem with an additional condition for the conjugate vector $\bar{\lambda}(t)$.

Recall that similar conditions hold for the optimal control problems with constrained state variables.

The obtained necessary conditions of optimality can be easily adjusted for a concrete variant of an estimation problem and for a given probability distribution of errors.

For the most common case of normal distribution $N(0, K_{\delta_i})$ of the vector $\bar{\delta}_i$ (with a zero average and the correlation matrix K_{δ_i}) we get the following boundary problem defining optimal estimate $\bar{x}_0$ of the initial state

$$\begin{gathered} \dot{\bar{x}} = \bar{\varphi}(\bar{x}, \bar{c}, t); \qquad \dot{\bar{\lambda}} = -\frac{\partial \bar{\varphi}^T}{\partial \bar{x}} \bar{\lambda} \\ \bar{\lambda}(t_0) = 0; \quad \bar{\lambda}(T) = 0; \\ \bar{\lambda}(t_i^+) = \bar{\lambda}(t_i^-) + \frac{\partial \bar{\psi}^T}{\partial \bar{x}} K_{\delta_i}^{-1}\{\bar{y}_i - \bar{\psi}[\bar{x}(t_i)]\}; \\ i = 1(1)N. \end{gathered} \tag{10}$$

Using the conditions of the theorem for estimates $\bar{x}_0$ and $\bar{c}$ we get the following boundary problem:

$$\begin{gathered} \dot{\bar{x}} = \bar{\varphi}(\bar{x}, \bar{c}, t); \qquad \dot{\bar{\lambda}} = -\frac{\partial \bar{\varphi}^T}{\partial \bar{x}} \bar{\lambda}; \\ \dot{\bar{\mu}} = -\frac{\partial \bar{\varphi}^T}{\partial \bar{c}} \bar{\mu}; \\ \bar{\lambda}(t_0) = \bar{\lambda}(T) = 0; \qquad \bar{\mu}(t_0) = \bar{\mu}(T) = 0; \\ \bar{\lambda}(t_i^+) = \bar{\lambda}(t_i^-) + \frac{\partial \bar{\psi}^T}{\partial \bar{x}} K_{\delta_i}^{-1}\{\bar{y}_i - \bar{\psi}[\bar{x}(t_i)]\}; \\ i = 1(1)N. \end{gathered} \tag{11}$$

Assuming correlation matrices in (10) and (11) to be a unit we obtain conditions for the varianional least squares methods.

IV. Comparison of Direct and Variational Approaches to Estimation

1. Let us consider a task of initial state estimation for $\bar{q} = \bar{x}_0$ and the normal distribution of errors.

In this case, the direct conditions of MLM are described via the system of normal equations

$$\bar{f}(\bar{q}) = \sum_{i=1}^{N} U^T(t_i, t_0) \frac{\partial \psi^T}{\partial \bar{x}_i} K_{\delta_i}^{-1}[\bar{y}_i - \bar{\psi}(\bar{x}_i)] = 0, \tag{12}$$

where $U(t, \tau)$ is a normalized fundamental matrix of the equation

$$\dot{\bar{\xi}} = \frac{\partial \bar{\varphi}(\bar{x}, t)}{\partial \bar{x}} \bar{\xi},$$

obtained via the linearization of system (1).

Using linearity of the conjugate system we get variational optimality conditions for estimates (10)

$$\bar{\lambda}(T, \bar{q}) = \sum_{i=1}^{N} V(T, t_i) \frac{\partial \psi^T}{\partial \bar{x}_i} K_{\delta_i}^{-1}[\bar{y}_i - \bar{\psi}(\bar{x}_i)] = 0, \tag{13}$$

where $V(t, \tau)$ is a normalized fundamental matrix of the homogeneous conjugate system

$$\dot{\bar{\lambda}} = -\frac{\partial \bar{\varphi}^T(\bar{x}, t)}{\partial \bar{x}} \bar{\lambda}.$$

Using the well-known properties of fundamental matrices of linear conjugate systems we can state (see [8]) the following equality:

$$V(T, t_i) = V(T, t_0) U^T(t_i, t_0).$$

Taking into account (12) and (13) we have

$$\bar{\lambda}(T, \bar{q}) = V(T, t_0) f(\bar{q}). \tag{14}$$

Now we see that variational conditions (13) hold only if the direct conditions (12) also hold. Hence, the estimates received for the both forms of optimality conditions will be equal.

2. Now let us compare calculation processes under the assumption that the Newton's method is used in both cases.

For the direct approach to a solution of equation (12) we have

$$\bar{q}_{k+1} = \bar{q}_k - \left[\frac{\partial \bar{f}}{\partial \bar{q}}\right]_k^{-1} \bar{f}(\bar{q}_k). \tag{15}$$

Calculation of the matrix $\frac{\partial \bar{f}}{\partial \bar{q}}$ in (15) can be carried out in two ways. The first one gives an approximate solution [1]-[4], [11], [12]. It ignores dependencies of $\frac{\partial \bar{x}_i}{\partial \bar{q}}$ and $\frac{\partial \bar{\psi}_i}{\partial \bar{x}_i}$ on the unknown value $\bar{q} = \bar{x}_0$ for derivatives of $\bar{f}(\bar{q})$ in (12); thus, only the dependencies on $\bar{\psi}_i$ and $\bar{q}$ are taken into account. In this case

$$\frac{\partial \bar{f}}{\partial \bar{q}} = R = \sum_{i=1}^{N} \left.\frac{\partial \bar{\psi}_i}{\partial \bar{q}}\right|_i^T K_{\delta_i}^{-1} \left.\frac{\partial \bar{\psi}_i}{\partial \bar{q}}\right|_i. \tag{16}$$

The second way provides strict values of $\frac{\partial \bar{f}}{\partial \bar{q}}$ according to t_j [13], [14]

$$\frac{\partial \bar{f}}{\partial \bar{q}} = R + \sum_{i=1}^{N} G_i. \tag{17}$$

The elements of the $n \times n$ matrix $G_i = [g_{j_r}^i]$ can be calculated as

$$g_{jr}^i = \sum_{p}^{N} \sum_{s}^{N} \left(d_{ps}^i\right)^{-1} \frac{\partial^2 \psi_s^i(\bar{q}_k)}{\partial q_j q_r} \Delta y_s^i,$$

where ψ_s^i and Δy_s^i are elements of $\bar{\psi}_s$ and $\Delta \bar{y}_s$ `;

$\left(d_{ps}^i\right)^{-1}$ are elements of the inverse correlation matrix $K_{\delta_i}^{-1}$.

It is obvious that a more strict evaluation of the matrix $\dfrac{\partial \bar{f}}{\partial \bar{q}}$ results in extension of a convergence area and raises the rate of convergence. Furthermore, if the matrix R is ill-conditioned then the use of (17) allows improving the conditionality (see [13]). However, the calculation of second-order derivatives implies computational complexity of greater order.

Variational approach to estimation comes to search of roots of equation (13). Thus, the computational scheme of the Newton's method is

$$\bar{q}_{k+1} = \bar{q}_k - \left[\frac{\partial \bar{\lambda}(\bar{q}, T)}{\partial \bar{q}} \right]_k^{-1} \bar{\lambda}(\bar{q}_k, T) . \tag{18}$$

However, the vectorial functions $\bar{\lambda}(\bar{q}, T)$ and $\bar{f}(\bar{q}, T)$ should meet the constraint (14).

The vector $\bar{f}(\bar{q}_0, T)$ is small for a good initial approximation $\bar{q}_0$. Hence, the following approximate equality follows from (14)

$$\frac{\partial \bar{\lambda}(T, \bar{q})}{\partial \bar{q}} = V(T, t_0) \frac{\partial f(\bar{q})}{\partial \bar{q}} . \tag{19}$$

Combining (14), (19), and (18), we approximately come to a computational scheme (15), where it is possible to obtain $\dfrac{\partial \bar{f}}{\partial \bar{q}}$ taking into account second-order derivatives of measuring values with respect to estimated parameters. Therefore, in terms of conditionality, convergence rate and area, the variational algorithms are not worse than direct ones using both the first-order and the second-order derivatives. Since the variational schemes do not need the computation of derivatives, an evident computational effect is granted. The fulfilled analysis and experiments confirmed that the computation speed of these algorithms was comparable with (and in many cases exceeded) the speed of the routine direct algorithms using only the first-order derivatives (16). Another advantage of variational estimation algorithms relates to simplicity of software.

V. Accuracy of Optimal Estimates

The accuracy of optimal estimates can be expressed via the correlation matrix $K_{\bar{q}}$. It is desirable to evaluate $K_{\bar{q}}$ through numerical computation of optimal estimations of the vector $\bar{q}$ without difficult additional calculations. Rao-Cramer's inequality provides the following approximation

$$K_{\bar{q}} = \Phi^{-1}(\bar{y} / \tilde{\bar{q}}) , \tag{20}$$

where

$$\Phi = -\frac{\partial^2}{\partial \bar{q} \partial \bar{q}^T} \ln W(\bar{y} / \tilde{\bar{q}}) = -\frac{\partial}{\partial \bar{q}} \bar{f}(\bar{y} / \tilde{\bar{q}}) ;$$

W is a likelihood function;

$\tilde{\bar{q}}$ is an estimation obtained via MLM.

Equality (19) is faithfully exact for the optimal estimations. Thus, using (20) and taking into account (19) we get the following formula for the calculation of $K_{\bar{q}}$

$$K_{\bar{q}} = -\left[V^{-1}(T, t_0) \frac{\partial \bar{\lambda}}{\partial \bar{q}} \right]^{-1} = -\left(\frac{\partial \bar{\lambda}}{\partial \bar{q}} \right)^{-1} V(T, t_0) \tag{21}$$

or

$$K_{\bar{q}} = -\left[U^T(T, t_0) \frac{\partial \bar{\lambda}}{\partial \bar{q}} \right]^{-1} . \tag{22}$$

The last formula is convenient for practical computing. The matrix $\dfrac{\partial \bar{\lambda}}{\partial \bar{q}}$ in (22) can be obtained for the optimal estimates of the vector $\bar{q}$. The matrix $U(T, t_0)$ can be easily determined through the finite difference method as a result of numerical integration of object dynamic equations simultaneously with the calculation of $\dfrac{\partial \bar{\lambda}}{\partial \bar{q}}$. Analysis and calculations confirmed that the estimations of the correlation matrix $K_{\bar{q}}$ were similar to estimations received via the linearization method.

VI. Orbit Determination by Measuring Results

Let us consider the application of MLM to statistical orbit determination by measuring current navigational parameters of motion of an artificial satellite. The measurements can be made at ground-based stations or on board.

Let us consider the equations of motion. We assume that the flight takes place in the normal gravitational field of the Earth (its oblateness is taken into account) at the enough altitude to make the influence of the atmosphere negligibly small. Here we use the absolute geocentric equatorial coordinate system providing equations convenient for programming [4].

$$\dot{x} = V_x ; \quad \dot{y} = V_y ; \quad \dot{z} = V_z ;$$
$$\dot{V}_x = -ax + F_x ; \quad \dot{V}_y = -ay + F_y ; . \tag{23}$$
$$\dot{V}_z = (2bc - a)z + F_z ,$$

where $\quad a = b[\alpha_{00} + c(d-1)] ; \quad b = R_0 r^{-3} ;$

$$c = 1.5\alpha_{20} R_0^2 r^{-2} ; \quad d = 5z^2 r^{-2} ;$$

$$J_{20} = -0.001082627 ; \quad \alpha_{00} = 62564951 \, m^2 / s^2 ;$$

$$\alpha_{20} = -67889{,}273 \, m^2 / s^2 ; \quad R_0 = 6371 \, km ;$$

$$r = (x^2 + y^2 + z^2)^{1/2} .$$

The functions F_x, F_y and F_z are projections of an acceleration caused by the influence of the atmosphere

$$\bar{F} = -S \rho \left| \bar{V}_{отн} \right| \bar{V}_{отн} ,$$

where S is the ballistic coefficient of a satellite depending on its mass, geometric and aerodynamic characteristics;

ρ is an atmosphere density depending on the altitude (standard models can be used);

$\bar{V}_{отн}$ is a velocity vector with respect to an approach flow (it is assumed that the atmosphere rotates with an angular velocity Ω)

$$V_{x_{отн}} = V_x + \Omega y ; \quad V_{y_{отн}} = V_y - \Omega x ; \quad V_{z_{отн}} = V_z .$$

Here we get the conjugate system

$$\dot{\lambda}_x = (a - Gx^2)\lambda_{V_x} - Gxy\lambda_{V_y} - Jxz\lambda_{V_z} ;$$

$$\dot{\lambda}_y = -Gxy\lambda_{V_x} + (a - Gy^2)\lambda_{V_y} - Jyz\lambda_{V_z} ;$$

$$\dot{\lambda}_z = -Jx\lambda_{V_x} - Jy\lambda_{V_y} + [a - 2bc(1-d) - Jz^2]\lambda_{V_z} ;$$

$$\dot{\lambda}_{V_x} = -\lambda_x; \quad \dot{\lambda}_{V_y} = -\lambda_y; \quad \dot{\lambda}_{V_z} = -\lambda_z; \qquad (24)$$

$$G = r^{-2}[3a + 2bc(2d-1)]; \quad J = G - 10bcr^{-2};$$

$$J_1 = Gz - 2bcdz^{-1}.$$

Thus, variational MLM reduces the orbit determination task to obtain unknown initial state vector $\bar{x}_0 = \bar{x}(t_0)$ via a solution of a two-point boundary problem of twelve nonlinear differential equations (23) and (24) with boundary conditions $\bar{\lambda}(t_0) = \bar{\lambda}(T) = 0$ and discontinuous change of the conjugate vector at the measuring points t_i

$$\bar{\lambda}(t_i^+) = \bar{\lambda}(t_i^-) + \frac{\partial \bar{\psi}^T(t_i)}{\partial \bar{x}} K_{\delta_i}^{-1}\{\bar{y}_i - \bar{\psi}[\bar{x}(t_i)]\}; \quad i = 1(1)N \quad (24)$$

The function $\bar{\psi}(\bar{x}, t)$ depends on the compound of measurements.

One more differential equation and boundary conditions should be added to (23) and (24) for obtaining the object initial state vector and the ballistic coefficient according to (11):

$$\dot{\mu} = -\rho\,|\bar{V}_{отн}|\,(V_{x_{отн}}\lambda_{V_x} + V_{y_{отн}}\lambda_{V_y} + V_{z_{отн}}\lambda_{V_z}),$$

$$\mu(t_0) = \mu(T) = 0$$

The analysis proved that an account of the air drag and the oblateness of the Earth in (24) did not result in an essential change in a problem solution. Hence, in many cases the following system can substitute for (24)

$$\dot{\bar{\lambda}}_r = \frac{\pi_0}{r^3}[\bar{\lambda}_V - \frac{3}{r^2}(\bar{r}\bar{r}^T)\bar{\lambda}_V]; \qquad \dot{\bar{\lambda}}_V = -\bar{\lambda}_r;$$

$$\bar{\lambda} = [\bar{\lambda}_r, \bar{\lambda}_V]^T = [\lambda_x, \lambda_y, \lambda_z, \lambda_{V_x}, \lambda_{V_y}, \lambda_{V_z}]^T;$$

$$\pi_0 = 398600{,}44\ km^3/s^2,$$

This follows from (99) under the condition of $\alpha_{20} = 0$ and agrees with the model of a central gravitational field.

VII. Outcomes of Experiments

The numerical results given below illustrate the use of the variational approach to orbit determination.

Computations were performed for a satellite in orbit of the altitude h=1000 km and the eccentricity e=0.003. The measurements of a current position were simulated at a given time interval with the fixed step of $\Delta t = 1\,s$. A random number generator was used for the calculation of errors fitting the normal distribution.

The received sample was regarded as a result of direct measurements through ground-based navigational facilities. The variational MLM method was used for data processing. The strict Newton's method and its simplified modification were applied to a boundary problem (23)-(25).

The considered variants of maximal measurement errors are enumerated in Table I.

The main goal of the numerical experiments was to investigate the convergence of the proposed estimating algorithms and the influence of errors upon the accuracy of estimates as compared with the "strict" values.

TABLE I

Variants of Maximal Errors

variant	$\Delta\bar{r}_{i_{\max}}$, *m*	$\Delta\bar{V}_{i_{\max}}$, *m/s*
1	100	0.5
2	150	1
3	200	2

The initial approximations $\bar{x}_0^0$ were chosen rather rough for examination of a convergence area. They differed from $\bar{x}_0$ by 500 km in coordinates and by 0.5 km/s in velocity projections. The iterations ended at Δr <1 m and ΔV <1 cm/s.

The experiments under these conditions showed that the strict Newton's method converged in 2 iterations on average. The speed of calculation rose up to 15% in comparison with the traditional direct MLM of first-order derivatives as the number of measurements increased. For the modified Newton's method the calculation time was half as large.

Typical stick-slip behaviour of conjugate variables corresponding to position and velocity vectors at integration of (24) is shown in Fig. 1 and Fig. 2. We can see that the functions definitely meet zero boundary conditions.

The dependencies of the results upon the sample size are shown in Fig. 3-6.

Mean square deviations of estimates are presented in Fig. 3 and Fig. 4, respectively.

Absolute errors (absolute differences from the "right" values) of position and velocity coordinates are presented in Fig. 5 and Fig. 6.

VIII. Conclusion

The variational MLM is an efficient addition to routine estimation methods to be applied to state determination in nonlinear dynamic systems and to navigational determination of motion parameters for spacecraft, in particular. Computational benefits of the proposed algorithms become apparent when the dimension of parameters and the number of observations grow, initial approximations become rough, etc. These algorithms can be applied independently or with direct (traditional) MLM to computation check, especially to reliable mass on-line estimations. They can be also used for the synthesis of measurement programs and for testing approximate algorithms.

Nowadays, variational MLM is developed rather intensively. New results were obtained in complex and additive estimation, in nonlinear dynamic filtration, sensitivity, measurement planning, etc. The details were omitted because of space limitation.

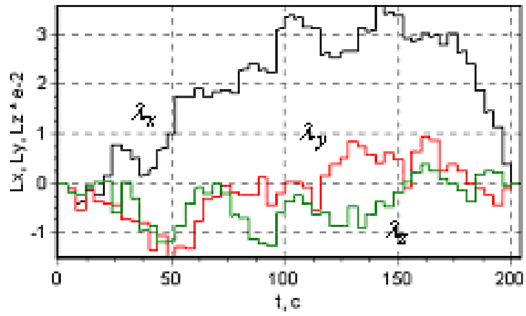

Fig. 1. Conjugate coordinates

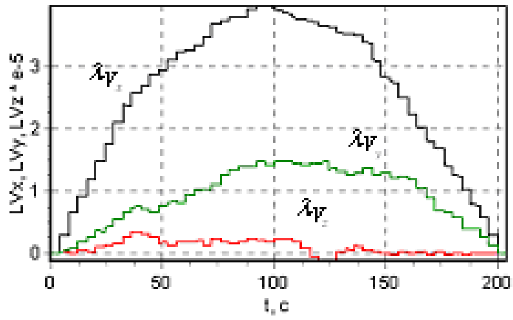

Fig. 2. Conjugate variables of the velocity vector

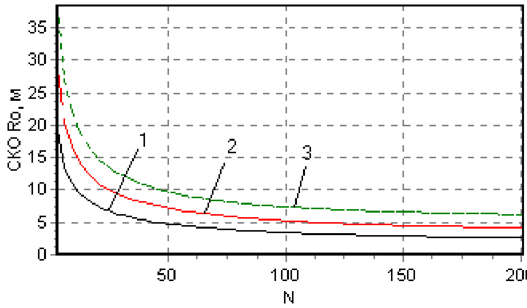

Fig. 3. Mean square deviation of coordinate estimates

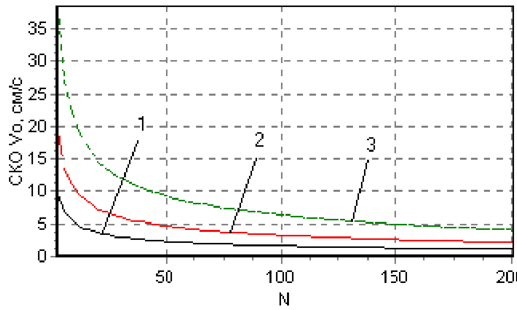

Fig. 4. Mean square deviation of velocity estimates

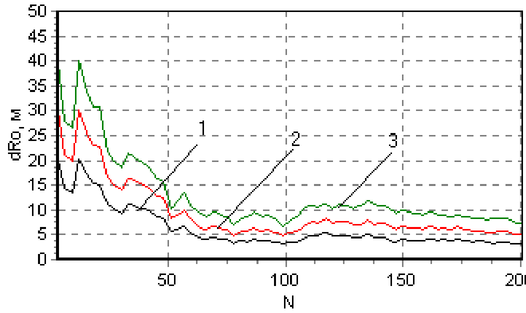

Fig. 5. Absolute values of coordinate errors

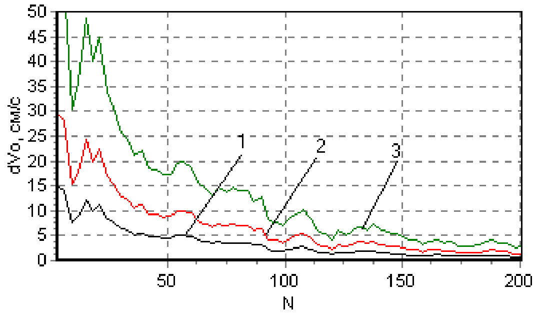

Fig. 6. Absolute values of velocity errors

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Vjačeslavs Mironovs, Jurijs Mironovs, Boriss Sokolovs, Rafaels Jusupovs. Maksimālās iespējamības metode un tās pielietošana aerokosmiskās navigācijas uzdevumos

Eksistējošās nelineāro dinamisko objektu kustības parametru statistiskās novērtēšanas metodes mērījumu nelineāro modeļu ar pilnu izlasi gadījumā sasniedza savu pilnību. Tās ir plaši aprakstītas literatūrā un ļauj risināt plašu klāstu svarīgu, sarežģītu lietišķu uzdevumu. Tomēr jautājumi par metožu un to skaitļošanas precizitātes pilnveidošanu joprojām paliek aktuāli. Rakstā piedāvātā metodoloģija balstās uz maksimālās iespējamības un mazāko kvadrātu metožu pielietojumu, kuras, kā rāda prakse, ir vienas no efektīvākajām kustības parametru noteikšanas metodēm. Kā zināms, maksimālās iespējamības metodes izstrādātājs ir R. Fišers, kas izstrādāja metodi statistiskā tipa objekta modelim. Tās izmantošanas procedūra paredz iespējamības funkcijas definēšanu un optimālā vērtējuma ieguvi, risinot normālvienādojumu sistēmu ar šīs funkcijas maksimizācijas nosacījumu. Pēc jēgas, normālai sistēmai piedāvā nepieciešamo optimalitātes nosacījumu, kas ir raksturīgs tiešas optimizācijas metodēm. Rakstā ir parādīts, ka, risinot dinamiskus novērtēšanas uzdevumus, minētajās metodēs tiek pielietoti tie paši vērtējumu optimalitātes tiešie nosacījumi, tomēr tie tiek definēti netieši, balstoties uz sākotnējo diferenciālvienādojumu sistēmu, kas apraksta objekta kustību, integrēšanas procedūrām. Parasti tiek novērtēts sistēmas sākotnējā stāvokļa vektors. Rakstā ir aprakstīts piemērs, kas ilustrē piedāvātās maksimālās iespējamības metodes dinamiskās interpretācijas lietderīgumu.

Вячеслав И. Миронов, Юрий В.Миронов, Борис В. Соколов, Рафаэль М. Юсупов. Метод максимального правдоподобия и его применение в задачах аэрокосмической навигации

Существующие методы статистического оценивания параметров движения нелинейных динамических объектов при нелинейных моделях измерений по полной выборке достигли высокого совершенства. Они широко освещены в литературе и позволяют, в принципе, решать широкий круг важных и сложных прикладных задач. Однако вопросы улучшения их точностных и вычислительных характеристик продолжают оставаться актуальными. Предложенная в статье методология в основном базируется на применении метода максимального правдоподобия и метода наименьших квадратов, которые, как показывает практика, являются одними из самых эффективных методов определения параметров движения. Как известно, метод максимального правдоподобия (ММП) изначально был разработан Р.Фишером применительно к модели объекта статического типа. Процедура его применения предусматривает составление функции правдоподобия и получение оптимальной оценки путем решения системы нормальных уравнений, выражающей условие максимума этой функции. По смыслу нормальная система представляет необходимое условие оптимальности, характерное для прямых методов оптимизации. В статье показано, что при решении динамических задач оценивания в указанных выше методах применяются те же прямые условия оптимальности оценок, однако, теперь они рассматриваются как заданные неявно на процедурах интегрирования исходной системы дифференциальных уравнений, описывающей движение объекта. Обычно оценке подлежит вектор начального состояния системы. В статье приводится пример, иллюстрирующий конструктивность использования предложенной динамической интерпретации ММП.